4-1-2010

Spring Design Optimization With Fatigue

John L. Porteiro
University of South Florida

Spring Design Optimization With Fatigue

by

John L. Porteiro

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Mechanical Engineering
Department of Mechanical Engineering
College of Engineering
University of South Florida

Co-Major Professor: Steven T. Weber, Ph.D.
Co-Major Professor: Rajiv Dubey, Ph.D.
Alex A. Volinsky, Ph.D.

Date of Approval:
April 1, 2010

Keywords: helical springs, design, stress, constraints, cartridge valve

DEDICATION

I would like to dedicate this paper to my father and mother, who are a constant source of inspiration and encouragement, without whom this document would probably not come to be. Their love and patience has nurtured and supported me through my many years, and I hope that I have done the same for them. I also acknowledge the various other teachers that I have had and who dedicated time to my growth.

ACKNOWLEDGEMENTS

I would like to acknowledge all those that encouraged me to write this thesis. I would also like to thank Dr. Steve Weber and Dr. Rajiv Dubey for their patience and knowledge in guiding me on this project. I am also grateful to Dr. Alex A. Volinsky for his helpful comments and careful review of the manuscript Along the way I also had the help of the engineering department of Sun Hydraulics, who were all very helpful and kind, providing valuable data and resources. The Spring Manufactures Institute, for providing research materials and basic information that forms the basis for much of the paper.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Optimization of Spring Design with Fatigue

John L. Porteiro

ABSTRACT

The purpose of this work is to look into the fundamental issues regarding spring design and develop a new, easy to use software program that would allow for optimal, flexible spring designs. Most commercial programs that address this function are basic and do not allow the designer much control over the variables hindering design. This is so because most programs start from the premise that the spring is a general purpose part of the system or that other design parameters can be altered to accommodate the chosen spring. In cases where this is not so, such as in hydraulic cartridge valves, where the geometric constraints are severe, spring design may be a cumbersome process. This is particularly true when fatigue life is taken into account.

The solution chosen here is to tailor the software application to these particular design constraints, incorporating some ideas about spring optimization. In addition to this, a concerted effort was made to make the subject more accessible to the engineers using the program by automating the more technical aspects of the process allowing the designers to make intelligent decisions based on how the variables would affect design. To this end currently existing software was evaluated to determine where it was lacking and a new program was written and painstakingly tested. Finally, it was used to correct flaws identified in existing springs.

CHAPTER 1

INTRODUCTION

1.1 What is a Spring?

What is a spring? According to A.M. Whal (1991): “A mechanical spring may be defined as an elastic body whose primary function is to deflect or distort under load (or to absorb energy) and which recovers its original shape when released after being distorted”. He goes on to classify the main functions of springs as one of four things: to absorb shock, to apply force, to support a structure, or to provide load control. This broad definition includes things that people do not normally think of as springs. Under this definition aircraft wings, the chassis of a car and even the shoes we wear will be considered springs. These items all distort under load and return to their original shape once the load is released. A shoe will absorb the impact of the foot fall and the bending of the arch of the foot and return to its normal state when these inputs are removed. Aircraft wings must take the loading and unloading of the wings on takeoff and landing in addition to any turbulence that the plane may encounter.

Obviously such springs have wildly different properties and functions, and so can not be all analyzed with the same techniques so, thus, the purposes of this paper, we will consider only helical compression springs (Figure 1). These springs are by far the most common type and are useful in the operation of many devices due to several desirable properties, such as a near linear rate (particularly after the first 20% of deflection), the many materials that can be used to make them, and the ease of manufacture. For this reason helical compression springs have been in use for some time.

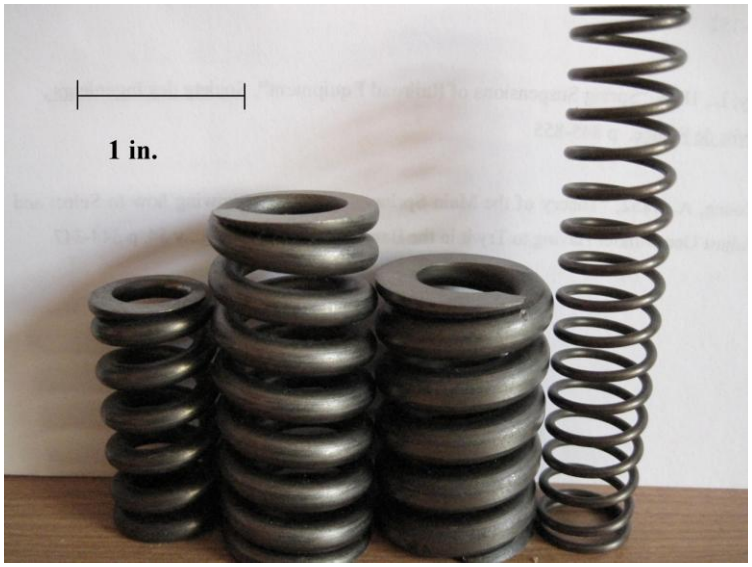


Figure 1 Compression Springs. Note that the end condition on these springs is that of closed and ground. You can see that the spring needs about a quarter turn to separate from the bottom coil.

1.1.1 The History of Springs

The history of springs is very long. No one can be sure when they were first created since they were probably part of some of the most basic tools. Relatively sophisticated devices, such as eyebrow tweezers have been dated as early as the Bronze Age. Ctesibius of Alexandria developed a process to manufacture springier bronze in the third century B.C. Example of military applications would be the bow and arrow as well as the more powerful catapult. More modern springs made of metals came about with the inventions of locks and clocks around the 15th and 16th centuries and were also used later in the suspensions for carriages (Figure 2). In the eighteenth century, the Industrial Revolution brought about methods, materials and techniques for the mass production of springs

(Figure 3). Springs today are made out of various materials and have different ways of achieving the same goals of storing and absorbing energy. Some late advances in spring technology include the development of springs made of plastic, Belleville springs, machined springs, and new nanotechnology developments that have resulted in springs that have the size of a molecule. The properties of a spring can be altered to suit the needs of a particular situation. A number of variables can be used to affect the properties of a spring. Among them we can include the way the ends of the spring are produced, the material the spring is made of, the type of surface treatment, and the design stress levels. All these variables can be used to alter the performance of the spring and a good designer must balance the variables to get the desired results.

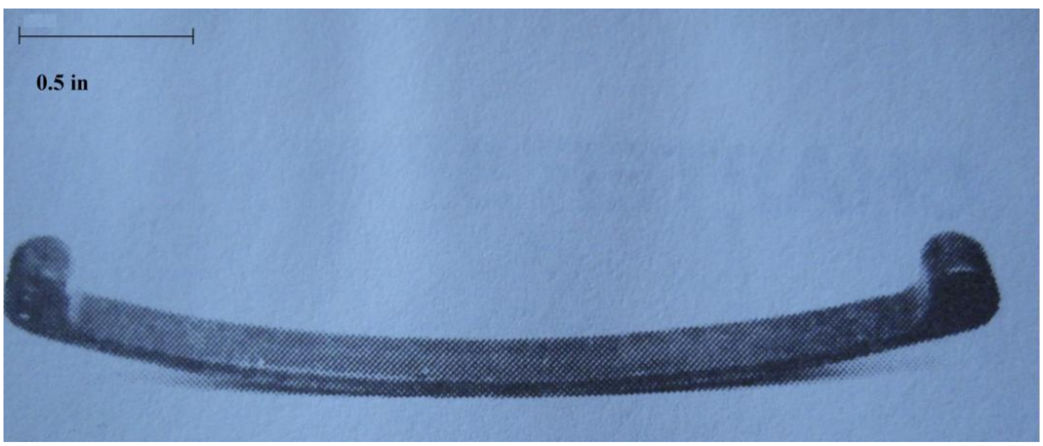


Figure 2 Leaf Spring (courtesy of Tubal Cain).

1.1.2 Current Spring Developments

Spring research carried out today involves the development of new materials in an attempt to create stronger, smaller springs. These efforts include the improvement of the basic materials currently used as well as the use of entirely new ones. Some advances in the technology of spring materials have resulted in the Stelmor process (controlling cooling of spring steel wires at the factory), the development of micro alloys (like SiCr) and in better quality controls procedures that allow a much better surface finish of the spring wires. An example of a new material is the use of carbon fiber springs in the suspension of race cars. As stated above, the advent of nano-science has resulted in the creation of springs the size of a molecule.

Taking advantage of the different material properties and types of spring these new developments have different ways of achieving the same goal of storing and absorbing energy; however, they are still springs that obey the same laws as the more traditional springs.

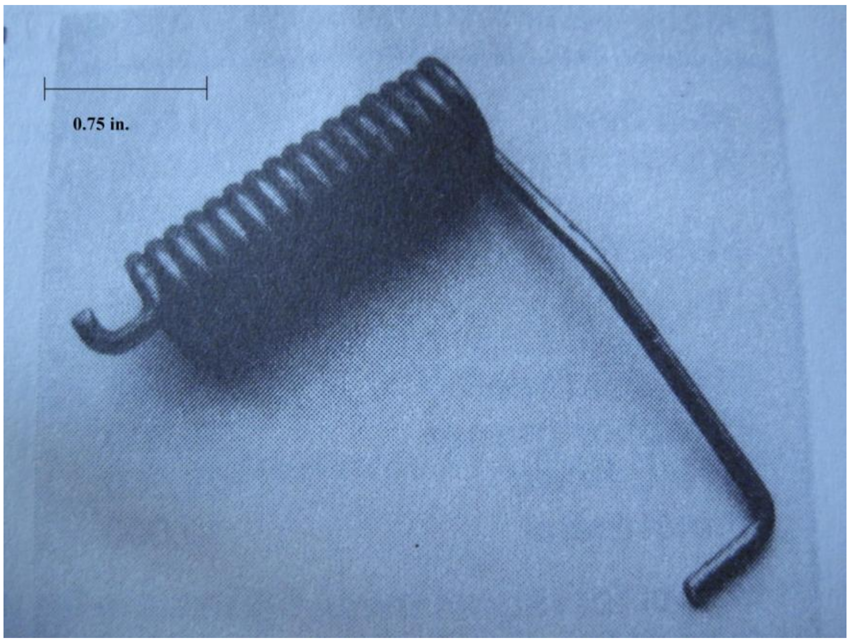


Figure 3 Torsion Spring (Courtesy of Tubal Cain).

1.2 Literature Review

Robert Hooke in his work “De Potentia Restitutiva” published in London in 1678 states “ut tensio, sic vis”, as extension so goes force. This simple statement, today known as Hooke’s law yields the basic equation for spring design. Two years later Lagrange (1770) analyzed the proportion of loads to deflection in springs. Practical applications of springs such as their advantages in improving the riding quality of carriages were discussed by Gilbert (1825) while the laws of isochronism of watches and chronometers was studied by Frodsham (1847). This last work also describes the design and materials used in spring watches. St. Venant (1847) studied the problem of torsion in rectangular prisms. In the following 50 years there was a tremendous growth in number of publications in spring design ranging from topics such as railway springs (Adams, 1850; Anonymous, 1858; Rey, 1876; Nadal, 1896; McCarty, 1898), watch springs (Young, 1852; Phillips, 1861; Caspari, 1875), Belleville Springs (Morandiere, 1866; Resal, 1888), spring motors (Doubler, 1876; Leveaux, 1876) and conical springs (Resal, 1892) among others. A description of Belleville springs and their advantages was given by Morandiere (1866) and Resal (1888) while the effects of steady and sudden loads on helical springs were investigated by Rankine (1866). The deflection and carrying capacity of spiral springs was studied by Begtrup (1892). French (1902) published formulas for the calculation of the torsional elasticity and safe stress levels for different ratios of wire to coil diameters. A. Roever (1813) proposed a pioneering theory for the description of the maximum bending and shear stresses on spring coils. He was the first to realize that the standard formulas for deflection and force exerted by springs were inaccurate because the stress distribution in the spring, due to pure torsion, did not have a linear relationship with the distance from the wire axis to the spring fiber. The so called “Roever effect” indicates that additional stresses appear due to the curvature, coil pitch angle and wire cross section, and a stress correction factor must be applied. For springs with coil to wire diameter ratios between 3 and 10, not using a correction factor underestimates maximum stresses between 14 and 30%.

Arthur Wahl (1929) proposed a stress correction factor to take into account those additional stresses using torsion theory and carried out tests that were in good agreement with the values theoretically predicted. Finniecome (1947) compared Roever's and Wahl's correction factors determining that for springs with coil to wire diameter ratios above 3 Wahl's values were slightly higher than Roever's. Honegger (1930) developed a new correction factor that included the displacement of the actual center of rotation of the fibers from the geometrical center of the cross section, carrying out tests to confirm his predictions. Goehner (1932) analyzed the maximum stresses in springs of circular and square cross section. The resulting formula was extremely complex and was simplified by Bergstrasser (1933). His results were compared with Goener's by Finniecome (1947) and found to be in excellent agreement. Wahl (1939, 1949, 1963) analyzed the use of the curvature correction factor in springs with large deflections, with free and fixed ends, under fatigue loading and endurance. An excellent analysis of when and when not to use the stress correction factor was presented by Carlson (1985).

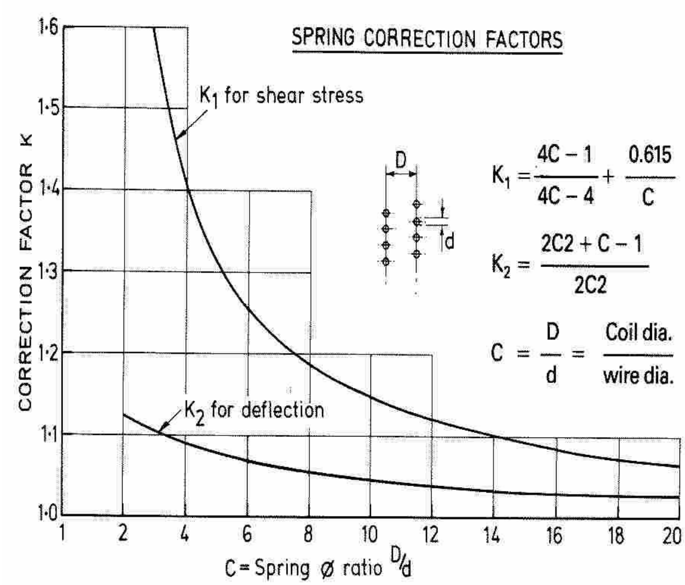


Figure 4 Wahl Stress Correction Factor (Courtesy of Tubal Cain)

Figure 4 shows the correction factor plotted out against the ratio of the diameter of the coil to the ratio of the wire diameter ($C = \frac{D}{d}$). This was the key innovation, in that the stresses are affected by the amount of curvature in the wire used to form the helix. The graph indicates that the stresses increase as C becomes smaller. This was found to be due to effects of curvature on the inner surface. The inner surface is shorter when compared to the outer surface and this means that it undergoes more deformation, increasing the stresses.

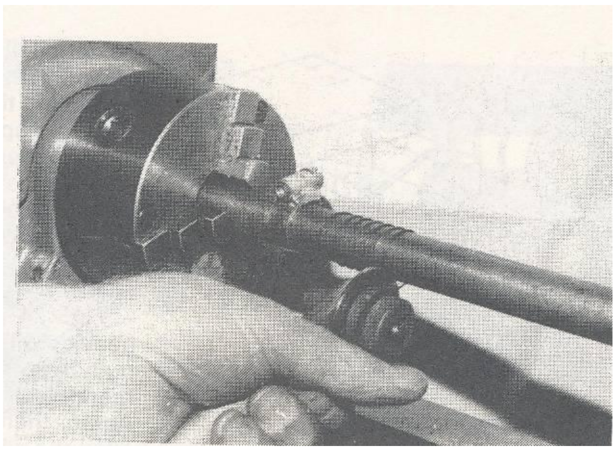


Figure 5 Manual Spring Coiler (Courtesy of Tubal Cain).

The spring industry has been a mature industry for some time now, still there is constant progress in all areas such as the studies in lateral deflection of free-end springs (Wolansky, 2005), the innovative application software for spring analysis (Zubek, 2010), the development and processing of new materials and the advent of newer faster more efficient automatic coiling machines that replaced the old arbor that was used for manual coiling (Figure 5)

1.3 Problem Statement

Any company that uses springs as part of an assembly must fit a spring into a specific space. In the initial design process this space can be altered to allow for the spring's requirements. However this can be very expensive later, when the product is in production. Often a product based on the original design is desired, butt may require a spring with different properties. An engineer is then tasked with re-designing a spring that must fit into the same space. Usually he will not have a particularly deep knowledge of springs, and so this can make the re-design more difficult than it should be. In the past the process consisted of calculating multiple designs and looking at charts to estimate their performance. This is a long and time consuming process. Furthermore, if the engineer is not experienced, it is possible to pursue a design that is feasible for the given space. This problem is particularly difficult when long life is needed. For long fatigue life, no suitable substitute exists for the chart method of design. To make such processes easier, an engineer needs some way to compare potential designs quickly with minimal knowledge of the underlying math and theory.

To date, there are programs that will calculate the properties of a spring given specific data. Some even include rudimentary fatigue life analysis. However there has been no effort to create a program that will help design a spring that would be a suitable substitute to an existing design, taking into account the fatigue properties. Such a program would have to look at several springs and use a set of criteria to suggest whether to accept or reject the spring.

CHAPTER 2

SPRING THEORY

2.1 Basic Models: Dimensioning

Basic spring theory states that a spring can be approximated by a (circular) bar in torsion. The derivation of this model can be seen in most text books on mechanics of solids and the twist in the wire is roughly equivalent to the deflection in the spring.

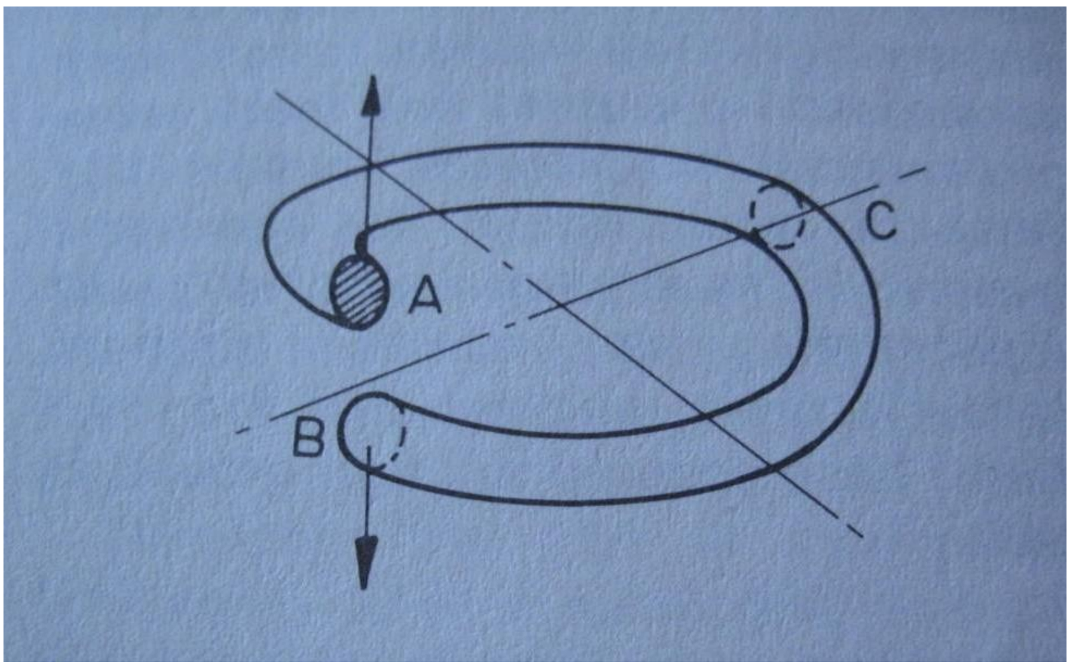


Figure 6 Simple Spring Model. The load from adjacent coils applied at A and B exerts a twisting force at C (with permission of Tubal Cain).

For an open coiled helical spring subject to an axial load W, the external work can be written as:

$$\frac{1}{2}W\delta = \frac{1}{2}T\theta + \frac{1}{2}M\phi \qquad [1]$$

where δ is the spring deflection, T and M are the torsion and bending moments, and θ and ϕ are the wire rotation and bending angles respectively. Substituting the values of T and M as a function of the load and α, the constant angle the coils make with the planes perpendicular to the axis we obtain:

$$\frac{1}{2}W\delta = \frac{1}{2}(WR\cos\alpha)\theta + \frac{1}{2}(WR\sin\alpha)\phi, \qquad [2]$$

or: $$\delta = R\left[(\cos\alpha)\frac{Tl}{JG} + (\sin\alpha)\frac{Ml}{IE}\right] \qquad [3]$$

where we have also substituted θ and ϕ by their values as functions of the bending and torsional moments as well as the elastic, and torsional modulus, (E, G) and the moments of inertia. We can write T and M as functions of α:

$$\delta = R\left[\cos^2\alpha\frac{WRl}{JG} + \sin^2\alpha\frac{WRl}{IE}\right] = WR^2l\left[\frac{\cos^2\alpha}{JG} + \frac{\sin^2\alpha}{IE}\right] \qquad [4]$$

Here, l is the wire length. For n turns of radius R its value is $2\pi Rn/\cos\alpha$. Substituting:

$$\delta = \frac{2\pi WR^3n}{\cos\alpha}\left[\frac{\cos^2\alpha}{JG} + \frac{\sin^2\alpha}{IE}\right] = \frac{4WR^3n}{r^4\cos\alpha}\left[\frac{\cos^2\alpha}{G} + \frac{2\sin^2\alpha}{E}\right] \qquad [5]$$

As a function of the coil and wire diameters, D and d, the expression is:

$$\delta = \frac{8WD^3n}{d^4\cos\alpha}\left[\frac{\cos^2\alpha}{G} + \frac{2\sin^2\alpha}{E}\right] \qquad [6]$$

When the angle α is small $\cos\alpha \cong 1$ and $\sin\alpha \cong 0$, leading to the standard form of the equation for deflection in a spring as seen here.

$$\delta = \frac{8WD^3n}{d^4G} \qquad [7]$$

This model applies to small angles of pitch, which are common in most springs. It does not account for the bending of the material, as can be seen from the derivation where bending is the second term. In most practical applications bending can usually be ignored. The correction factor developed by Wahl compensates for the added stresses due to the curvature absent in the torsion bar model of the spring and for the effects of direct shear. Figure 7 shows the stress distribution with the Wahl correction factor. There is also a correction for deflection but it is seldom used due to the small effect that it has on the deflection (for most springs around 3%). These are the most important attributes of springs, other attributes like the diameter of the spring, the spring wire, and the number of coils (or turns) in a spring, will also be discussed.

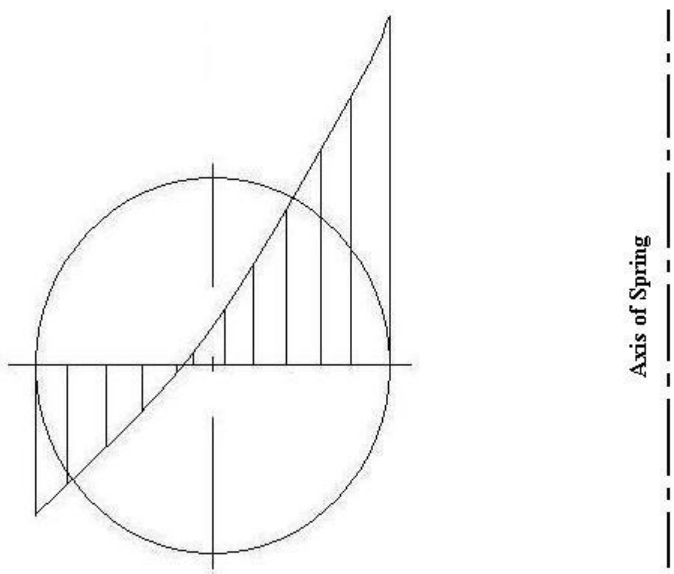


Figure 7 Schematic of the Shear Stress Distribution in a Wire Cross Section with Wahl Correction Factor.

The diameter of a spring and the wire diameter are related in that they affect the same properties. Increasing the diameter of the coil by a factor of two will half the spring rate, (the ratio of load to deformation). Doubling the wire diameter will result in an eight fold increase in spring rate. One of the consequences of this is that a spring cannot be scaled down and maintain the same properties. Consequently the wire diameter and the diameter of the coils have been found as useful in describing the springs properties when combined as C (the ratio of coil to wire diameter). This ratio is used in several equations including Wahl's correction factor. Springs usually fall in a range of C between 3 to 20. Springs with a low C (around 4) are very stiff and suffer from greater effects due to curvature, while higher values (20 or so) tend to buckle under load.

Increasing the number of coils also alters the spring rate. It does so because each individual coil deflects a certain amount under load (stress) independently of the other coils. This is because the load is distributed throughout the spring wire evenly. Thus the more coils present the more the spring deflects. For most materials it is desired to maintain a number of coils that keeps the pitch fairly low. This increases stability and more uniformly distributes a load. This can be seen in the equation for eccentricity of loading:

$$\frac{e}{r} = 1.123(Z - 1) \qquad [8]$$

$$Z = 1 + \frac{0.5043}{n} + \frac{0.1213}{n^2} + \frac{2.058}{n^3} \qquad [9]$$

Where:

$n =$ Number of solid (total) turns of wire.
$e =$ Distance of the load axis from the spring axis.
$r =$ Coil radius

The more coils in the spring the closer to unity the load will be.

The distribution of load also affects the fatigue life, so the number of coils is more important than it is generally assumed.

One final variable that affects the number of active coils is the “end condition” of the spring. This refers to the way the spring makes contact with the surfaces around it. There are four basic conditions: open, closed, open and ground, and closed and ground. The open condition occurs when the wire is cut to the desired length of spring and no change in pitch or special machining is made in order to make the spring more stable. This condition would be used if there is a special fitting designed to hold the spring in place. Open and ground is similar to the open condition with the exception that the end is ground flat so that when placed on a surface the wire end surface will be parallel to it. A closed end is such that the last coil is decreased in pitch so that the two last coils are touching. This allows for a more stable spring that will stand on its own without any special fitting. The closed and ground ends are similar to the closed, but in this case, it is ground flat, which allows the spring to be much more stable than the other types and to stand up level with the ground.

The closed end types are desirable because of the stability against buckling that they provide, but in order to do so, extra material is used that will not deflect. In addition the closed end conditions introduce a non-linear spring rate at the beginning of deflection since the coils close to the ends do not have full pitch to deflect and will touch the end coil sooner, becoming inactive. In Figure 1 all springs except for the one on the far right have about 2.5 coils inactive due to the end coil condition (1.25 per end).

Figure 8. Fatigue Failure of a Spring.

2.2 Fatigue Design

The fatigue life is an important part of any design and springs are no different. Springs are often in service in critical applications where failure is unacceptable. Since the most popular material for springs is steel, the appropriate fatigue calculations use metals as the material. Metal fatigue is the failure of a component as a result of cyclic stress below the yield stress. The failure occurs in three phases: crack initiation, crack propagation, and catastrophic failure. The duration of each of these three phases depends on many factors including fundamental raw material characteristics, magnitude and orientation of applied stresses, processing history, ambient conditions and excitation frequency. Fatigue failures often result from applied stress levels significantly below those necessary to cause static failure (elastic regime).

On a microscopic scale, failure occurs along slip planes in the crystalline structure of the materials. Most metals with a body centered cubic crystal structure have a characteristic response to cyclic stresses. These materials have a threshold stress limit below which fatigue cracks will not initiate. This threshold stress value is often referred to as the endurance limit. In steels, the life associated with this behavior is generally accepted to be $2x10^6$ cycles. In other words, if a given stress state does not induce a fatigue failure within the first $2x10^6$ cycles, future failure of the component is considered unlikely. For spring applications, a more realistic threshold life value would be $2x10^7$ cycles.

Metals with a face centered cubic crystal structure (e.g. aluminum, austenitic stainless steels, copper, etc.) do not typically have an endurance limit. For these materials, fatigue life continues to increase as stress levels decrease; however, a threshold limit is not typically reached below which infinite life can be expected. In steels the cause of this endurance limit is generally attributed to the presence of interstitial elements such as carbon or nitrogen that pin dislocations preventing the slip mechanism that leads to micro cracks (Bannantine, 1990). Endurance limits are affected by the conditions and will disappear if the metal element is subjected to periodic overloads, corrosive environments, or elevated temperatures.

The charts used to predict the fatigue life of a metal component are based on empirical data from a large numbers of tests and provide us with the most reliable method for fatigue life prediction. The stress life or S-N (Stress – Number of cycles) method is one of the preferred methods and it has been in use for over 100 years. The S-N approach is valid in applications where the stress stays within the elastic region, and life expectancy is long. As a result, this method will not apply to static loading conditions or situations where the life expectancy is less then 1000 cycles as this tends to involve the plastic region. This is due to the fact that the stress life approach ignores the true stress-strain behavior treating all strains as elastic. Fortunately the S-N curves for different steel alloys are similar and they allow the use of a single curve, modifying the results to account the difference in elastic modulus. To estimate the S-N curve for steel the following power relation was found (Bannantine, 1990).

$$S = 10^{C} N^{b} \quad \left(for\ 10^{3} < N < 10^{6} \right) \tag{10}$$

where S is the cyclical stress and N the number of cycles. If S_e and S_u are the endurance and ultimate stresses and S_{1000} the stress after 1000 cycles:

$$b = \frac{1}{3} \log_{10} \frac{S_{1000}}{S_e} \tag{11}$$

$$C = \log_{10} \frac{\left(S_{1000}\right)^{2}}{S_e} \tag{12}$$

If one makes the assumption that $S_{1000} \approx 0.9 S_u$ and $S_e \approx 0.5 S_u$ the preceding equations reduce to:

$$S = 1.62 S_u N^{-0.085} \tag{13}$$

This assumption only works for steels as for other materials S_{1000} and S_e are not as clearly defined.

Because it is nearly impossible to predict when an individual component will fail one must rely on a statistical analysis and safety factors to account for extreme cases. The data for the analysis has to be obtained from many tedious and expensive tests. Many engineers have looked for an empirical relationship between the applied stress and the life of the spring and over the years many such relations have been proposed and found to be useful:

Gerber (Germany 1874): $\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_u}\right)^2 = 1$ [14]

Goodman (England 1899): $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u} = 1$ [15]

Soderberg (USA 1930): $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = 1$ [16]

Morrow (USA 1960): $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{\sigma_f} = 1$ [17]

Where: $\sigma_a = \frac{\sigma_{\max} - \sigma_{min}}{2}$ is the stress amplitude. [18]

$\sigma_m = \frac{\sigma_{\max} + \sigma_{min}}{2}$ is the stress mean. [19]

All of these relations connect the endurance limit on the alternating stress (stress amplitude, σ_a) axis with the yield strength (S_y), ultimate strength (S_u), or true fracture stress (σ_f), on the mean stress axis(σ_m). Using the alternating and mean stresses is one of the more convenient ways to represent fatigue loading conditions. The Soderberg relation is considered too conservative and is seldom used. The actual test data falls between the Goodman and Gerber lines, with Gerber being closer to the results and at times overestimating, and Goodman being more conservative. For most situations the mean stress is small compared to the alternating stress and there is little difference in the theories. When the ratio of maximum to minimum stress is close to one the theories are distinct; but this is a region for which little data is available. Figure 9 shows these various plots in a chart.

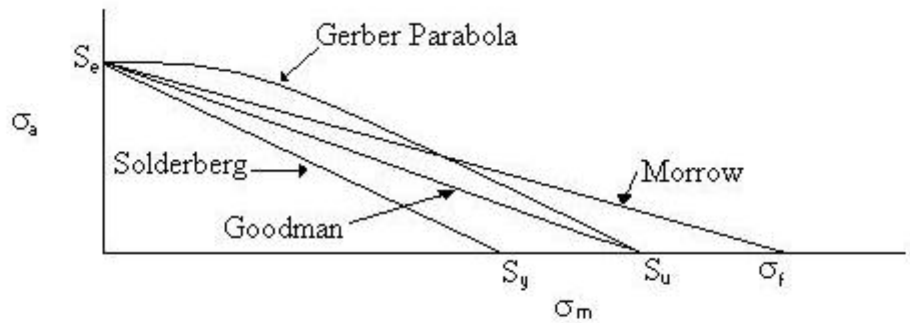


Figure 9 Comparison of Goodman, Gerber, Soderberg and Morrow Lines

The cyclical stresses are usually used to create a Goodman diagram. In this diagram the alternating stresses are on the y axis and the mean stresses are on the x axis. The value for the endurance limit is then placed on the alternating stress axis and the ultimate tensile strength on the mean stress axis. These are then connected with the Goodman line (infinite life). A line drawn from the origin with the slope of the alternating stress to the endurance limit is the load line. To find the life of the spring one finds the point located buy the mean stress and the alternating stress (this should be on the load line). Figure 10 shows a basic Goodman diagram.

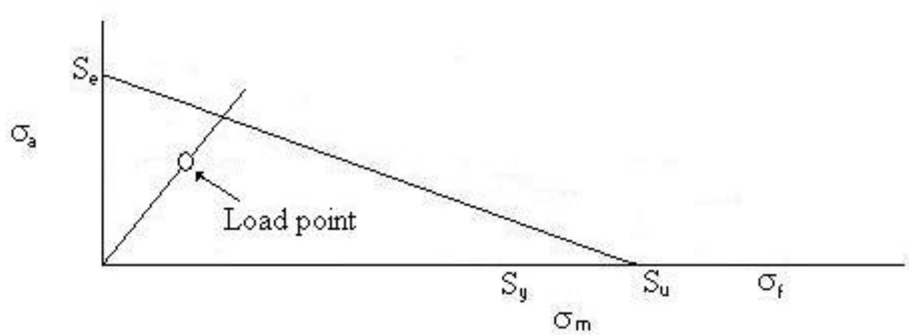


Figure 10 Goodman Diagram

Another popular way of predicting life is the modified Goodman diagram. It is different in that it is usually normalized so that it covers a range of similar materials. As stated in the Spring Design Manual of the American Society of Mechanical Engineers (Warrendale, 1996, p.90) "to construct a modified Goodman diagram the initial and maximum stresses are normalized by dividing them by the ultimate tensile strength of the material", ($\text{Initial stress} = \frac{\sigma_{\text{inital}}}{S_u}$, $\text{Maximum stress} = \frac{\sigma_{\max}}{S_u}$). This allows the diagram to be used for various similar materials with different tensile strengths. Starting at the initial load point a vertical line can be drawn. Along this line are the various loading ratios that the spring can achieve. The data from S-N curves is usually incorporated as life lines at the top of the graph showing expected life. This allows for the inclusion of some common surface treatments such as shot peening, because shot peening does not reduce stresses, like presetting, and so will not alter the stress values. The modified Goodman diagram presented in Figure 11 includes an S-N curve showing the effects of shot peening as well as an S-N curve without shot peening.

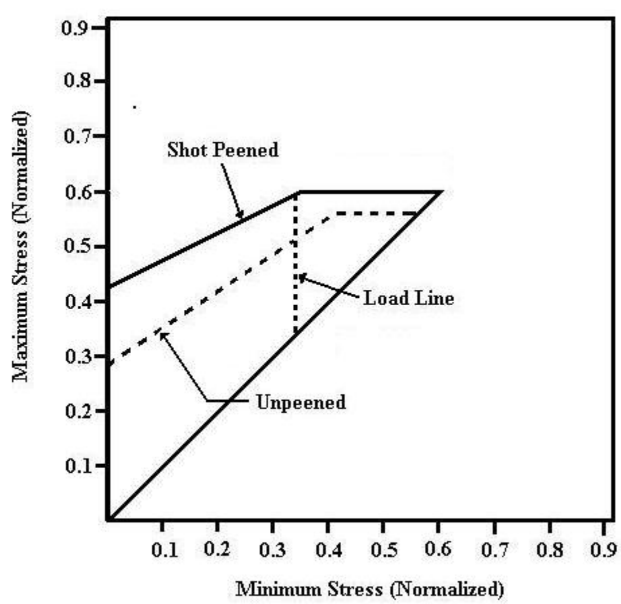


Figure 11 Modified Goodman diagram

The manner in which a spring is stressed also affects its fatigue life. Springs that are cycled over a narrow stress range (10,000 psi) generally have long life, and if the range is small enough (5,000 psi) the stress correction factor can be ignored (Carlson, 1969). As noted above, the load is seldom distributed evenly and it has been found that springs that do not have whole numbers of active coils will have longer fatigue lives as these distribute the loading stresses better (Carlson, 1969). From this, it is easy to see that lateral loading of a spring (as in a vibration damper) will also reduce its fatigue life. The models for predicting spring failure cannot be generalized to all springs because of the many factors that affect it, therefore specific conditions must be used to get an accurate prediction. There is no "general" model that can predict fatigue life (Carlson, 1969).

Unfortunately stress in a spring is not uniform. The stress is greater on the inner surface and the coil surface itself is vulnerable to imperfections in the material that serve as stress concentration points. These inevitably lead to cracks that cause catastrophic failure in a spring, reducing its effective fatigue life. For this reason fatigue failures always start at the inner surface and proceed outward (Carlson, 1969). To prevent cracks from forming and spreading, spring makers have found several solutions: protective coatings, surface treatments after the spring was coiled, and treatments to induce residual stresses in the spring that would be beneficial.

An example of surface treatment is case hardening, which makes the surface harder to scratch, and more resistant to cracks. This is done by heating the material and allowing the grain structure on the surface to reorganize, changing its mechanical properties. Shot peening is an example of inducing residual stresses control.

Shot peening is a process where a small round sphere or other object is propelled with enough velocity to create local yielding on the surface (a small dent). This local yielding causes compressive stresses to be built up on the surface around the impact area. Once this stress is there, it will automatically act to close or prevent a small crack from spreading on the surface. It has been shown that any process that creates compressive stresses on the surface of the spring will increases the life of the spring.

Surface coating can be as simple as painting the spring protecting it from the environment and potential crack-producing abrasions or corrosive chemicals, or it can be as complex as electroplating.

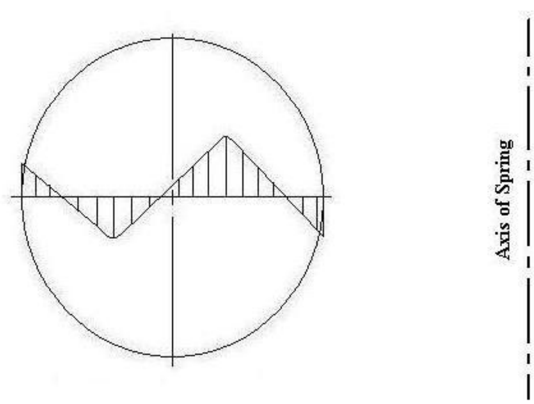


Figure 12 Residual Shear Stress Distribution After Presetting.

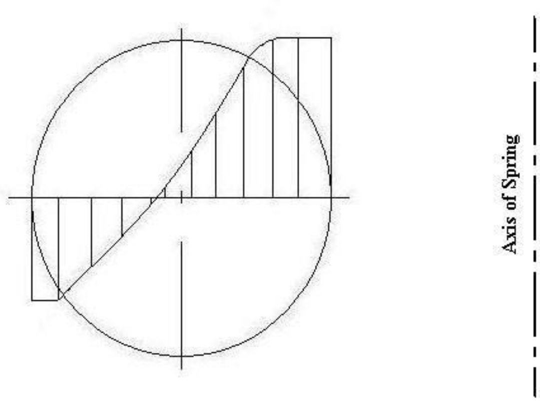


Figure 13 Loaded Shear Stress Distribution After Presetting.

These various processes are all useful in increasing the life of springs, and are usually used in combination. Shot peening and heat treatments are particularly useful, where plating is more difficult to achieve because the bond between the two metals must be vary strong to prevent separation during torsion of the material. A typical spring might go through the following processes: after coiling and annealing (to remove stresses from the coiling process), a dip in anti corrosive liquid, a pre-set to induce beneficial residual stresses, and shot peening. Figure 12 shows the residual stresses that are induced by pre-setting. Figure 13 shows the new stress distribution when the spring is loaded. All these work together to preserve and enhance the fatigue life of the spring. When properly done this can increase the stress ranges up to thirty percent or increase the life up to ten fold (Carlson, 1969).

2.3 Spring Design Due to Existing Constraints

The springs that are considered here are for pre-existing shapes that can not or would be prohibitively expensive to be altered. As such there are limits to what can be done to the spring design. The object then is to find a way to extract the needed force out of the spring while maintaining the same form factor. One helpful approach is minimize the use of resources to allow for the best utilization of the available space. In an article in "*Springs"* Henry Sweiskowski (1995) explains that there are three basic ways to minimize the amount of material. The first uses the initial load as the variable for optimization. He states that retainer springs (lock washers) are in this group. The second optimizes the final load. The author did not specify which type of spring this group was best suited for but statically loaded springs seems to be a good fit. The third method is the optimization of the energy stored for a given working stroke. He commented that springs used for stopping or accelerating a mass belong in this group. This case is the one that interests us the most but all are useful. The core of this work involves taking the derivative of the appropriate variables (wire diameter, coil diameter and spring index) in the spring equations to minimize spring volume. Of the three cases, we will derive the third case as it is useful for our purposes. We start with five equations:

$$\frac{P}{F} = \frac{Gd^3}{8D^3 n} \qquad [20]$$

where $\frac{P}{F}$ is the spring rate (load, P over deflection, F)

the stress at the final load is :

$$S_2 = \frac{8DP_2}{\pi d^3} \qquad [21]$$

the working stroke is :

$$s = F_2 - F_1 \qquad [22]$$

the energy storage can be found from :

$$E = \left(\frac{P_2 + P_1}{2}\right) \qquad [23]$$

and the volume of the spring material used is :

$$V = \frac{\pi^2 d^2 D n}{4} \qquad [24]$$

We can now combine these equations and solve for the volume:

$$V = \frac{\pi^2 s^2 d^6 G}{8D(\pi s S_2 d^3 - 8ED)} \qquad [25]$$

Making the substitution $C = \frac{D}{d}$:

$$V = \frac{\pi^2 s^2 D^4 G}{8C^3(\pi s S_2 D^2 - 8EC^3)} \qquad [26]$$

Taking the derivative of the volume with respect to the spring index and setting it to zero will yield the minimum volume:

$$16EC^3 - \pi s S_2 D^2 = 0 \qquad [27]$$

Solving for C:

$$C = \sqrt{\frac{\pi D^2 S_2 s}{16E}} \qquad [28]$$

replacing C with $\dfrac{D}{d}$ gives

$$d = \sqrt{\frac{16ED}{\pi S_2 s}} \qquad [29]$$

Finally if [29] is substituted for into [26] we obtain

$$V_{\min} = \frac{4EG}{S_2^2} \qquad [30]$$

Sweiskoski also notes that when Eq. 29 is used to choose the wire diameter, the minimum value for volume is independent from the coil diameter and stroke. It is also worth noting that Eq.30 can be solved for the energy stored (E) showing the direct relation between the torsional modulus and the energy capacity of a spring, while solving for S will shows that the stress is proportional to the square root of the volume of material. This may be part of the reason for the push for better materials in springs rather than new theories and design techniques.

There has always been a drive in for smaller springs that are able to store more energy, as can be seen from one of the first uses of springs in the pocket watch. Research in this area continues to this day, although most of it deals with new alloys or different processes after winding to allow the spring to withstand greater stresses, or last longer. Despite this long history, it is hard to find information about maximizing the amount of energy in a given volume or for a given deflection and the majority of programs that exist only calculate the spring rate and deflection based on parameters submitted by the user.

The energy is stored in the spring essentially as stress. The desired fatigue life is often the limiting factor when designing for optimum efficiency. This means that a wire thicker than the optimum value is often needed in order to keep stresses within acceptable levels.

Central to the ideas of spring design just examined is the contention that there is more than one configuration that will achieve the desired results, with one the most desirable solutions being that the amount of material used to be a minimum. To find the optimum spring design one could reason that the design that stores the maximum energy, *U*, for a given space and material would be the best. But as we see when deriving the equation for stored energy, the energy stored is essentially the stress on the spring. This means the design is also constrained by the maximum shear stress, f_s that the material can support:

$$U = \frac{f_s^2}{4G}\frac{\pi^2 d^2 Dn}{4} \qquad [31]$$

$$f_s = Maximum\, shear\, stress$$

The amount of energy stored by the spring can be increased by adding material, increasing *d*, *D* and *n*, or increasing its maximum shear stress, by changing materials or various treatments after coiling. New alloys provide more strength and thus more energy storage but at a greater cost and often with other limitations as well. Extra processes aimed at increasing the life or strength of a material also add cost and complexity to a spring. For this reason it is difficult to know which design will be cheaper as the design may minimize the amount of material, and then a host of processes could be used to improve its basic capabilities at added cost, a different material and different processes could be used to also produce an optimal design. With this in mind the engineer must contribute to the final decisions, but the process can be made easier by speeding up the spring design.

CHAPTER 3

OPTIMUM SPRING DESIGN WITH FATIGUE

3.1 Proposed Model

Current commercially available spring design programs allow for the determination of spring parameters typically as a function of free length, coil diameter, spring rate and spring material. Other parameters such as number of coils, stress factors, wire diameters etc. are chosen by the program and not available to the designer as a design variable. In a good number of cases this is undesirable because this makes it difficult to find acceptable designs when volumetric constraints are present such as in hydraulic cartridge valve spring designs.

In developing the model, our main objective is to address this problem by optimizing the design of a spring using a greater number of chosen spring parameters in a way that requires a minimum of knowledge and experience from the user. The calculation procedure that the program uses automates the process of trial and error to select a spring that will satisfy the design requirements. This saves time and allows the designer to eliminate many designs that would not work and reduces design time. The program incorporates a subroutine designed to expedite this process. The subroutine is not meant to produce a final spring design as it is not possible to evaluate whether the spring will satisfy all the criteria that the designer has in mind, instead it produces a design that satisfies all major design criteria and constraints and is a robust starting point to a final customizing process. In contrast, the models that are currently available in most programs will calculate a spring once the designer has specified enough criteria. This is despite the fact that some of the criteria like the number of coils, the diameter of wire, the diameter of the spring coil, and the free length, are almost never critical and can be varied in almost all designs. This process allows the engineer greater flexibility in designing a spring and allows greater customization than currently available.

3.1.1 Spring Variables and Constraints

Most spring design programs assign equal importance to all variables. Because of that, variables that are important to most designers such as spring rate and free length are given the same weight as variables that are not as important to the design such as the wire diameter and the number of coils, which are seldom important factors in designs. Another parameter of secondary importance is the coil diameter which can usually be varied somewhat without significantly affecting spring performance, so that it can be increased or decreased to suit a particular design. By acknowledging these facts, the designer will find it much easier to create a design that will work. Most engineers do not have experience that would allow them to realize this. This becomes critically important when the fatigue life is included in the design. This difficulty is due to the relationship between physical constraints that affect the design and the ability of the designer to correctly anticipate operating load conditions. A spring can be designed such that given the same critical properties: spring rate, free length and approximate coil diameter, it will have different fatigue life because of the variation of other parameters. This fatigue life varies directly with the amount of material in the spring in general, more material means longer life. As noted previously, the most common constraints to designs are free length, spring rate, and coil diameter. Free length is important because most spring designs try to avoid having any type of gap between the spring end and the load, because a gap would cause an impact loading condition. This would reduce fatigue life and require a greater safety margin. In addition, such impacts usually affect the part in which the spring is installed causing unforeseen stresses and vibrations. Increasing the length to eliminate a gap without changing the number of coils will also change the spring rate. The number of turns is altered to allow the spring to reach the new length and to maintain the same rate. Because all the coils deflect the same amount for a given load, if more coils are added the result is a larger deflection. Since the spring is longer, it needs more deflection to keep the same deflection to length (rate) ratio. If the number of coils is kept constant (so the pitch is altered to reach the new length), the rate will remain the same. Altering the number of coils is not usually done in commercially available programs, because the general spring theory applies only to small variations in pitch and it becomes possible to violate this condition when the spring pitch becomes large. When the pitch is large, the individual coils can deflect a greater distance before the spring goes solid (the condition when the spring is fully deflected and there is space between coils). If so, the material can be subject to plastic deformation due to excessive stresses. This will cause a permanent set, lowering the original free length and greatly reducing fatigue life.

Spring rate is one of the most obvious design parameters. As it is usually the most important property of a spring, it is almost always specified by the designer. A spring with a spring rate that is too weak for the given load will compress until solid. When this point is reached the spring is simply deforming the material and not storing energy nor can it absorb impact. If the spring rate is too high the spring will not deflect, it will effectively behave like a solid bar and will be of little use. Spring rate is affected by wire and coil diameters and to a lesser extent by the number of coils (or turns). So altering these allows the engineer to change other spring properties while maintaining the same rate. In addition to this, spring rates are affected by the tolerances of the design and the quality of the material used. Because of this, springs usually vary by about ± 3% from the calculated values.

The number of coils, although completely irrelevant to the designer most of the time, is not a user set parameter in most commercial programs. This variable is usually dictated by the mechanics of the spring. As noted previously it affects the spring rate, but it is not used to alter the design. The risks of reducing the number of coils are: overstressing (possibly taking a permanent set) and going solid. It can, however, be used in applications where there is a large available deflection before the spring becomes solid. Also there are secondary effects that would push a designer to look for designs with a specific number of coils (related to load distribution). These effects are generally small and can be ignored in most designs.

Coil diameter is more useful as a variable than the number of coils, but it is not one that can be greatly altered. Changing the coil diameter will affect the stresses and the spring rate, but less so than changing the wire diameter. It also has the advantage of a wider range of available values (unlike wire diameter) and it will not affect the available deflection making it less complicated to use as a variable. The coil diameter is limited by the fact that springs usually have to fit in a cavity or over some other object (sometimes another spring). Designs must give the spring a clearance between it and its housing and any object that it may fit over. This somewhat limits the designer’s ability to alter the coil diameter and its usefulness as a variable.

The variable that is most useful in terms of design is the wire diameter. It has a strong impact on the attributes of a spring. Despite the limited number of wire diameters available, this is something that seldom hinders a design. Altering this value has to be done carefully. Increasing the wire diameter will result in slightly lager corrected stresses and less deflection. From a practical point of view, the wire diameters that can be used are limited to the standard sizes available from the distributors. Changing the diameter can sometimes alter the strength of the material as is the case with music wire. This means that it is often necessary to make adjustments to the coil diameter to achieve the rate that is desired.

3.1.2 TK Solver

The author used TK solver to develop and run the spring design program. TK Solver is a powerful modeling software program that uses a rule-based language and a robust engine for solving systems of equations, either algebraically or by iteration. It can skip equations for which it needs information, and continue on to others. It then returns to the bypassed equations and tries to solve them now with the new information. This is done via multiple passes through the “rule sheet”, referred also as the main sheet, containing all the equations that the program will use. This sheet can include subroutines and procedures, but these are not treated in the same multiple pass system; rather they are solved line by line. The subroutine codes are complete in that loops and function calls and error messages within the program can be used, much like in other programming languages. A screen shot of the program is shown in Figure 14.

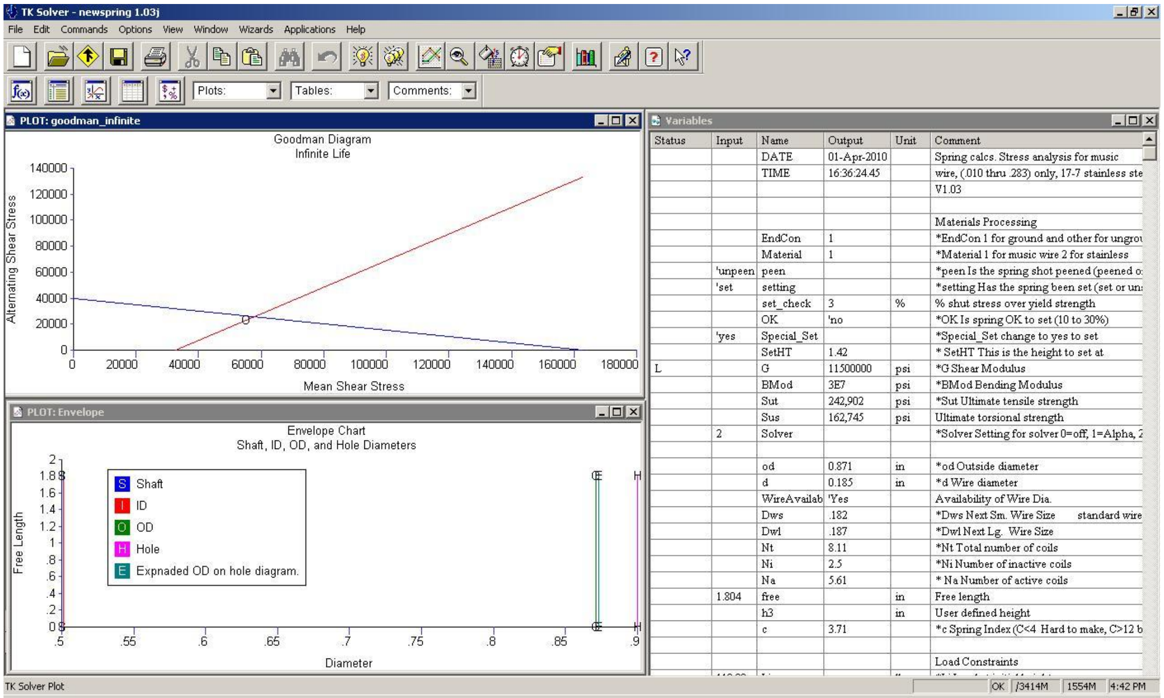


Figure 14 Screen Shot of Software Program

3.1.3 Numerical Procedure

The numerical procedure is one of iterative redesigns, that is, the design parameters obtained in one iteration can be reapplied as starting parameters of the new spring design. Once this is done and the new criteria selected, the program then goes about finding a solution that meets the criteria set forth. Some of the criteria are internal and kept within the program. These criteria are related to stress levels and how close to solid the spring will be once loaded to the determined height.

The program requires four variables to generate a spring design. An additional five, four for fatigue and one for the buckling condition, are required to determine the full fatigue life. The program can solve for any variables, but it will generate an error if the coil diameter is not one of the parameters given. This is due to the rules in the program. When the solver subroutine is used, the wire diameter no longer needs to be specified.

The two settings of the solver subroutine, as well as its internal procedures are of interest so they will be examined in detail here. On the first setting the solver tries to find a spring that is at lest 30% below the stress needed to make the spring yield. It also looks to use 85% of the available deflection. These values were chosen to avoid the problems of non-linearity that occur from the spring going solid. The encyclopedia on spring design by the Spring Manufactures Institute states “when it is necessary to specify a rate it should be specified between two test heights which lie within 15 to 85% of the full deflection range”. These values allow the spring a good margin of safety of operation. The program also works to make the spring stay below 85 % of the maximum fatigue stress. The process that the solver uses is a nested loop. The solver starts by using the given outer diameter, subtracting a tenth of an inch from it, and choosing the smallest available wire size. It then checks the spring to see if it satisfies the criteria that the designer has in mind. If the criteria are not met, it increases the coil diameter by a thousandth of an inch. By increasing the coil diameter, the spring rate and stress will be lowered. This means the higher stress solutions will be eliminated first. When the coil diameter has been increased by two tenths of an inch, the inner loop will exit and the wire diameter will be increased by one size. Then the inner loop starts again. The inner loop is where the checks for fatigue life and working load are done. If a solution is found both loops are ended. From this point, the designer can turn the solver off and make small adjustments to the design that better suits his purpose. The initial design is usually not usable for practical reasons. Because spring manufacturers will provide coil sizes in quarter of a turn increments only, a case when the program specifies a spring with a number of total coils that does not conform to this fact is not acceptable. Because of this, it is important to be able to fine tune the design.

The basic variables used by the program, initial and final load heights, spring rates, free length, wire diameter and outer coil diameter (or inner) are interchangeable. When using the solver subroutine the wire diameter cannot be specified and an initial guess of the coil diameter must be supplied.

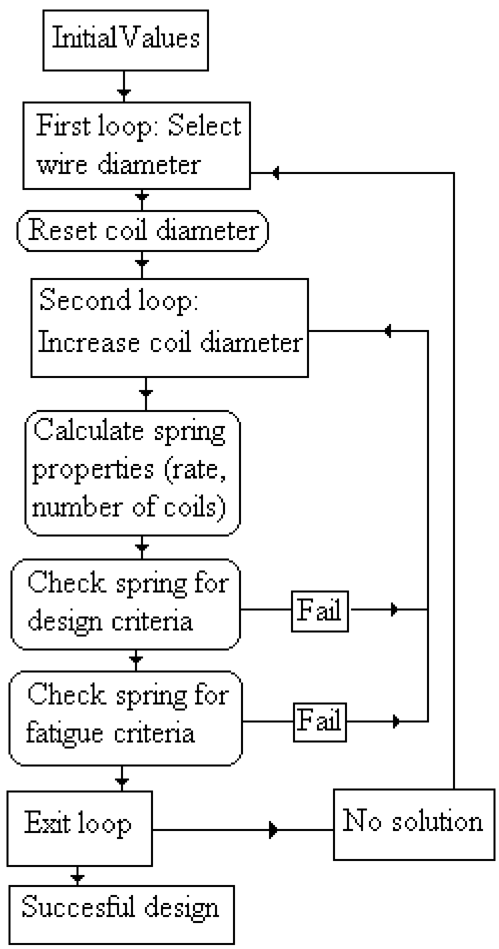


Figure 15. Program Flow Diagram

Following the flow diagram shown in Figure 15 we will go through an example assuming that the free length, initial and final working loads, and initial load height are specified. The buckling condition will be that of one end pivoted and one constrained, which is a common constraint for springs. After the program initializes the variables, it enters the first loop. This loop is set to cycle through every wire diameter available. It does this by taking the number of wire sizes in the internal list and using this as an index or counter for the number of cycles. The loop counter is used to select the wire size from the list. It then calculates the maximum amount that the diameter can be varied. It does this by using the hole and shaft parameters, subtracting the shaft diameter from the hole. This value is then divided by the variation increment (set to 0.001) and this is the number of cycles for the second loop. Before entering the second loop, the initial diameter of the spring is set. This is done in the first loop since the diameter must be reset each time the second loop is completed. It is set to the shaft diameter plus twice the wire diameter just selected. Once this is done the second loop is entered.

In the second loop, the spring is actually calculated. First the initial and final working heights are subtracted to obtain the change in height. Then the initial and final loads are subtracted and divided by the change in height to calculate the spring rate. Once the rate is obtained, the number of active coils is calculated using the shear modulus (G), the wire diameter, the rate, and coil diameter. The solid height is then calculated using the number of turns and wire diameter. Then the solid height is checked to see if a solution is possible with the current wire diameter. After this, various factors are calculated including basic working stresses at initial, final, and solid loads. Then the program calculates the Whal correction factor, the spring index, and the ultimate tensile strength. There is a check performed to see if a spring can be set (compressed to solid to induce beneficial residual stresses). Then the yield stress is calculated based on whether the spring is set or unset. The program has two different setting, each one of them using three criteria to finalize the design. For the reasons stated previously, the first setting looks for a spring that uses at most 85% of the available deflection, is 15% above solid height and uses 85% of the yield strength. The second setting uses at most 90% of the available deflection, is at least 10% above solid height and uses 90% of the yield strength. If the spring can meet the three chosen criteria then the program proceeds to calculate the fatigue properties of the spring. The fatigue section calculates a large number of parameters related to the fatigue life of the coil. Among them are the mean stress, alternating stresses, fatigue life and effects of shot peening. Then a final check is made and if the spring passes this check it sets the exit variable to complete and the loops are both exited. The program then returns the new wire diameter and the diameter of the coils.

3.2 Numerical Results

To test and verify the program and its capabilities three Sun Hydraulics springs, Springs 238, 225 and 239 were used as a test for the program.

Spring 238 was one of the better documented springs having two tested revisions. The second revision was necessary because one of the first samples tested failed (yielded) under load. The original design of Spring 238 was flawed in that the spring rate was too high, with a spring rate around 970 lbs/in, and it was intended to fit over another spring, 239, which had a diameter of 0.515 inches, not leaving any clearance. Studying the basic information about it provided in Table 1, it is obvious that the spring had no fatigue life problems since without presetting or shot peening it still had an infinite safety factor of

1.13, and was still 15% above the yield stresses at the final loading point. This indicates that shot peeing is probably unnecessary and was added more as an additional safety margin. In the first redesign (see Table 1) the wire diameter was reduced to 0.177 inches

Table 1: Spring 238

	Original	First Redesign	Second Redesign
Outer Diameter [in.]	0.889	0.875	0.900
Inner Diameter [in.]	0.515	0.521	0.526
Wire Diameter [in.]	0.187	0.177	0.187
Total Number of coils	8	7	7.75
Number of active	5.5	4.5	5.25
Rate [lb/in]	932	930	924
Free length [in.]	1.815	1.804	1.835
Solid height [in.]	1.496	1.239	1.449
End condition [1]	Closed & ground	Closed & ground	Closed & ground
Initial Load [lb.]	118	118	118
Final working height	1.688	1.677	1.707
Initial stress [lb/in^2]	46,327	53,300	46,800
Final load [lb.]	234.00	234.00	234.00
Final stress [lb/in^2]	1.563	1.551	1.581
Final working height	92,060	106,300	92,800
Percent below yield	50.17	3.45	49.49
Estimated life	Infinite	<100K	Infinite
Infinite safety factor	1.85	0.92	1.96

the in order to allow for a larger inner spring clearance. The diameter of the coil was then constricted to 0.875 inches to maintain the spring rate. The free length and the inner diameter were shortened to 1.804 and increased to 0.521 inches, respectively, to allow the spring to achieve the load targets and to fit over the inner spring. The number of coils was also reduced to 7 from the original 8 (increasing the spring rate at the same time). As can be seen, these measures increased the stresses placed on the spring. Three samples of the first redesign of Spring 238. The high stresses caused one of the springs to fail during testing and although the other two springs performed well in terms of spring rate, a detailed examination of the test data led to the conclusion that this design was not acceptable because it was almost yielding at the final load.

The initial redesign of Spring 238 was made without the aid of the fatigue calculations. After the failure, it became apparent that this was not practical and, as a result, fatigue calculations were added to a later version of the program. Running the program again for this spring with the added fatigue calculations yielded a fatigue life value that indicated that the spring was suitable for static loading only. The spring life could be improved by setting, which would boost the life to over 100 thousand cycles, but this would require a special setting apparatus since compressing it to solid (or shut), the usual way of setting springs, would cause it to deform or break. The yield height was just under a tenth of an inch lower than the final load (with a corresponding load increase of about 90 lbs). Shot peening would also increase the fatigue life and with the special set would give the spring a calculated infinite safety factor of 1.48. In addition, in this redesign the yield point was extremely close to the final working height, so it was decided to try a second redesign.

For the next redesign the wire diameter returned to its original size (0.187 inches) and the outer diameter of the spring was increased to 0.900 inches. The number of active coils was also slightly reduced by a quarter turn. This allowed the spring rate to remain as needed while reducing the stress levels. The free length was actually increased in this design, as there were no clearance issues with the adjoining part. Because of this the spring was given a tolerance that tended to reduce the free length $\left(1.835 \pm {}^{0.000}_{0.010}\,\text{in}\right)$.

The problems with this spring that were related to the spring rate were corrected and the final spring had a slightly lower spring rate of 924 lbs/in while maintaining a calculated infinite safety factor above 1.5.. The initial and final working heights are a little higher then the original design but that was found to be acceptable.

Table 2: Spring 225

	Original	Redesign
Outer Diameter [in.]	0.656	0.636
Inner Diameter [in.]	0.392	0.386
Wire Diameter [in.]	0.132	0.125
Total Number of coils [1]	7.5	7.5
Number of active coils [1]	5.75	5.00
Rate [lb/in]	527.5	526
Free length [in.]	1.250	1.250
Solid height [in.]	0.990	0.937
End condition [1]	Closed and ground	Closed and ground
Initial Load [lb.]	70	70
Final working height [in.]	1.117	1.117
Initial stress [lb/in^2]	45,700	52,300
Final load [lb.]	111	111
Final stress [lb/in^2]	72,500	83,000
Final working height [in.]	1.039	1.039
Percent below yield stress [%]	52.9	46.57
Estimated life [cycles]	Infinite	Infinite
Infinite safety factor [1]	2.52	1.52

The springs that follow were redesigned once and tested. They exhibited problems related to spring rate or the number of active coils. The next spring to be redesigned was the 225 spring, shown in Table 2. This spring also had a rate problem. Measurements showed a rate of 610 lbs/in while the design called for a rate of 527 lbs/in. The design had a calculated infinite safety factor of 2.52 when set and shot peened, so fatigue life was not an issue. One of the problems that were identified was that the number of inactive coils (or turns) was inaccurate at 1.75 inactive coils. The closed and ground end condition means that at least one coil is inactive at both ends. In addition, because of the helix angle, the spring does not separate from the ground coil until about a quarter turn. This adds up to about 2.5 inactive coils. The fewer coils deflecting caused the spring rate to be higher than desired.

Another problem was that the spring was very close to solid when the final load was applied, leaving 0.05 inches of deflection. This forced the spring maker to be very careful with the tolerance on the wire diameter as typical manufacturing tolerances of the wire could cause trouble.

The redesign decreased the coil diameter and reduced the wire diameter, while correcting the number of active coils to 5. The decreased wire diameter allowed the spring to deflect more, easing the fears of the spring going solid because of an out-of-tolerance wire. Also, the calculated load height is exactly the same as in the original design.

The largest change is the fact that this spring has a lower fatigue life safety factor, and this is in part because the spring is not shot peened in the design. However, it is set and without this, the safety factor drops to 1.45. For the case in which the spring is shot peened and pre-set, the factor of safety jumps to 2.52. This did not seem necessary and because of the added cost to the spring manufacture, it was removed.

Spring 176, shown in Table 3, the next to be studied had similar problems to Spring 225 in that it had an impractical value for the number of inactive coils. This also caused the spring to have a higher than calculated spring rate value (284 lbs/in).

Table 3: Spring 176

	Original	Redesign
Outer Diameter [in.]	0.462	0.470
Inner Diameter [in.]	0.272	0.272
Wire Diameter [in.]	0.095	0.093
Total Number of coils [1]	9.5	10.25
Number of active coils [1]	8.75	7.75
Rate [lb/in]	271	264
Free length [in.]	1.313	1.313
Solid height [in.]	0.903	0.953
End condition [1]	Closed and ground	Closed and ground
Initial Load [lb.]	NA	NA
Final working height [in.]	NA	NA
Initial stress [lb/in^2]	NA	NA
Final load [lb.]	67.5	67.5
Final stress [lb/in^2]	104,000	90,500
Final working height [in.]	1.062	1.058
Percent below yield stress	14.1	45.52
Estimated life [cycles]	<100 K	100+ K
Infinite safety factor [1]	0.63	.86

This spring has only three quarters of an inactive coil turn and this is not possible for the end condition given since for a closed spring one needs at last one full turn to form the spring base at both top and bottom. In addition, another quarter turn at each end is needed for the spring to reach its full pitch. Because of this, the spring rate is too high. Although the infinite life factor was not listed as one of the problems, the spring as designed was calculated as below infinite life. These calculations were done without shot peening or presetting since that is what was specified for the spring. One of the reasons the safety factor is so low is that the spring is using almost all of the available deflection causing a very severe loading condition.

The redesign corrected the coil problem and some other modifications were made, namely adding shot peening and setting to improve fatigue life. The improvement in fatigue life was still not enough to give it infinite life as per specifications the spring rate was also lower. The coil diameter was slightly increased and the wire diameter decreased to 0.93. This had to be done as an increase in coil diameter would make the spring go solid. The spring had to gain an inactive coil and lose an active one. This makes the total active coils 7.75 and the total number of coils 10.25. This resulted in a solid height of 0.953 inches, still a reasonable distance from solid but not as much as the original. The infinite life factor of 0.86 is better than the original design, but due to constraints, the wire diameter cannot be increased. Increasing the wire diameter would improve the fatigue life of the spring.

The next spring we will look at is Spring 239. It also had problems with the number of inactive coils though not as much as Spring 176. This means that its rate was also off. Another concern was that the final working height was close to making the spring go solid. Of interest is that this spring was used inside the previously examined spring 238.

Looking at the solid height and the final load height, it is apparent that only 0.03 inches separates them. The fatigue life on this spring is high enough so that even without shot peening the spring will have infinite life safety factor of 1.33, although presetting of the spring must be done since the spring is so close to solid at its final working height.

The redesign decreased the number of inactive coils and again reduced the wire diameter. The spring rate is slightly lower, and the working heights are closer but a little higher then the original. This is due to an increased free length. The details of the spring are shown in table 4 below.

The wire diameter was decreased to 0.095 and the coil diameter reduced to 0.498 inches. The free length was increased to allow the rate to be closer to the original. This spring was also shot peened and set, since without shot peening the infinite life safety factor drops to 1.11. The fact that this spring is not quite as durable should come as no surprise since all of the modifications increased the stress levels.

Table 4: Spring 239

	Original	Redesign
Outer Diameter [in.]	0.510	0.498
Inner Diameter [in.]	0.315	0.308
Wire Diameter [in.]	0.100	0.095
Total Number of coils [1]	9.5	8.5
Number of active coils [1]	11.5	11.0
Rate [lb/in]	212	210
Free length [in.]	1.487	1.517
Solid height [in.]	1.150	1.045
End condition [1]	Closed and ground	Closed and ground
Initial Load [lb.]	36.8	36.8
Final working height [in.]	43,100	49,200
Initial stress [lb/in^2]	1.313	1.342
Final load [lb.]	63.3	63.3
Final stress [lb/in^2]	74,100	84,700
Final working height [in.]	1.188	1.216
Percent below yield stress [%]	53.96	47.85
Estimated life [cycles]	Infinite	Infinite
Infinite safety factor [1]	2.16	1.89

3.3 Comparison with Test Results

The test results were compiled using a compression tester that produced a graph of the load versus the deflection curve. The tester had two load cells with 100 lb and 1000 lb ratings. The springs were compressed at a rate of half an inch per minute. To compensate for the flex of the machine a calibration curve was created for both load cells and it is available in the appendix.

The raw data was then turned into a graph. This is accomplished by selecting points on the load-deflection curve and placing them into Excel. The points are then used to calculate rates by taking the first and second pairs of data and subtracting them. This gives the change in load and the change in distance. Then the change in load is divided by the change in deflection yielding the rate. This process is repeated with second and third points and so on until the number of desired points is accumulated. This raw data is further altered by using the calibration curve to compensate for the deflection of the tester.

The graph shows the spring rate as a function of the applied load. One thing that will be obvious from looking at the graphs is that springs are not linear in general, but they are very close to being so.

The spring rate changes the most at the initial deflection as some coils close to the ends go solid, and when the spring is near solid height as all the coils start to touch and go solid. The calculations then reflect an average value, not any specific value. The graph below (Figure 16) shows the spring 238's two redesigns and compares them. the first redesign has two data sets, one with a spring that did not yield during testing and the other that did. The spring that yielded is easily picked out as the one that shows a dramatic drop in the spring rate when it reaches the end of the test. The point at which it failed was the final load height specified in the spring design. The rate for that spring drops about 50 lbs/in when the other two samples increase by that amount. As expected the two first revision springs have very similar loading curves, and an average spring rate of 891 and 895. This rate is a little low when compared to the 930 lbs/in the design calls out. The second load curve is not as linear but it has an average spring rate of 921 lbs/in, closer to the 927 lbs/in calculated in the design.

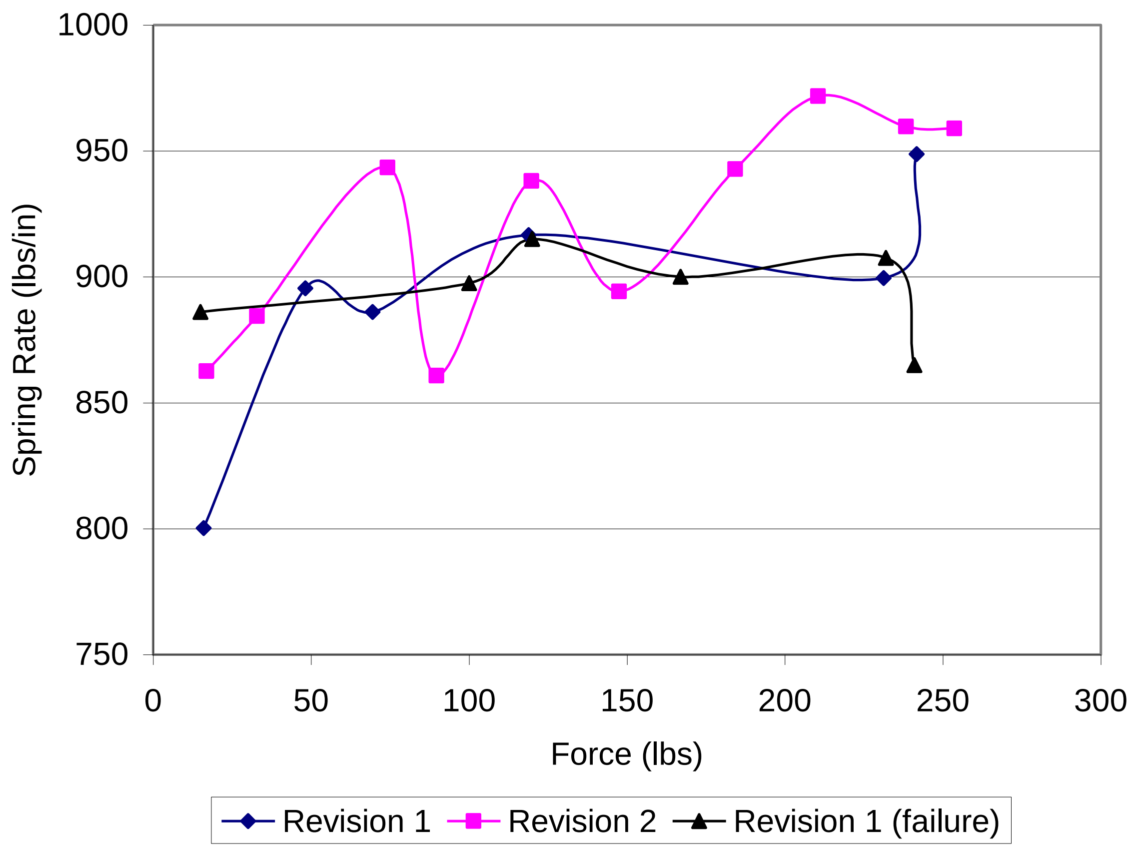


Figure 16. Spring 238 Redesigns

Spring 238's redesigns are fairly consistent. First redesign was a weaker than expected, but it was the most linear of the groups that was tested. This makes it unfortunate that it would fail as it did. The second redesign is quite erratic in that the data points that were calculated seem to have fluctuating rates.

Spring 225 had a predicted spring rate of 527 lbs/in close to the 526 lbs/in that was predicted. In addition the spring seams to have a good working region that has a linear spring rate as can be seen in the graph (Figure 17). The initial jump in rate is not

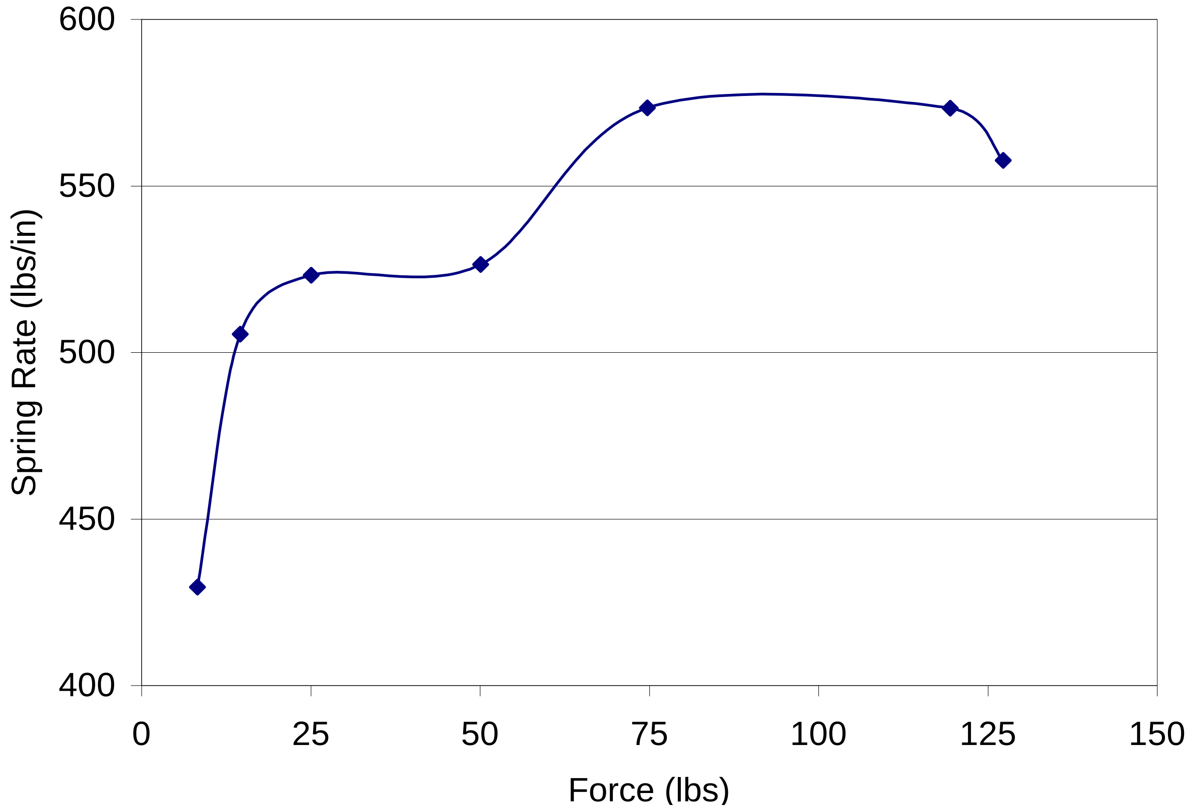


Figure 17 Spring 225 Redesign.

uncommon as the coils near the ends will very quickly touch the end coils, causing the spring rate to jump up as fewer coils deflect. Though there is a dip at the end this was not due to yielding. It is remarkable that the spring rate is so close to the predicted value, and some of the other springs that were tested did not have rates this close but they were in the range of 550 lbs/in which is still a fairly accurate prediction.

Spring 239 was designed to fit inside spring 238. It was redesigned to achieve a spring rate of 210 lb/in. The test results are shown in Figure 18.

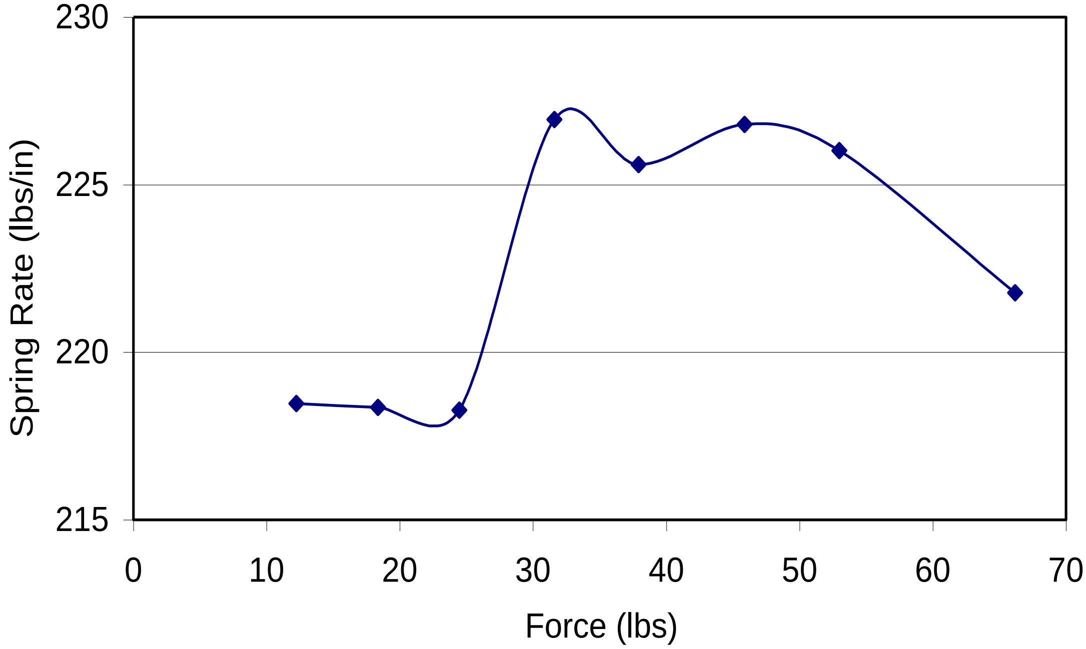


Figure 18 Spring 239 Redesign

The test data shows that the spring rate is very linear, with an average spring rate close to the design value with a narrow range, less than 5% within the testing range. It was therefore considered a successful redesign.

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS

4.1. Conclusions

The program was successful in modifying existing spring designs producing springs that, when tested, were found to have spring rates equal, within tolerances, to those calculated by the program. Multiple springs were tested and so the program can be considered reliable. Fatigue life calculations were incorporated and proved successful. No spring that was redesigned by this program had problems in terms of fatigue life when tested.

The addition of automation makes the process of spring design easier on the designer. In addition, the program's internal criteria for selecting springs such as checking for how much of the deflection is in the linear region of the spring, how close the spring is to yielding, and what the fatigue life of the spring is, prevent bad designs from being suggested. This reduces the level of expertise required to run the program, making it more accessible. This is important as most engineers do not have much experience designing springs, yet may be required to do so to incorporate a spring into a design.

The freedom to alter any parameter of the spring allows the designer to create designs that other commercially available software will not. The automation makes it easier for the designer to use the program, but it does so without removing the customizability of the spring design, allowing the designer to make adjustments as needed. This for example means that the number of coils in a calculated design can be altered to allow the designer to provide a specification which a manufacturer can reliably produce.

The process of spring optimization depends ultimately on what the designer whishes to optimize. Most designers prefer designs that minimize use of resources and thus the expense of producing the design. This is a complex issue and the flexibility of the program allows the designer chose how to go about it. The program provides a solution that minimizes the use of materials by using the smallest wire and coil diameter for a given fatigue life, it then allows the user to optimize the spring to suit his other needs.

4.2. Recommendations for Further Work

The program would benefit greatly from a graphical user interface. This would allow the user to see more clearly the relevant data. Currently this information is only available in a worksheet. This detracts from the ease of use, and makes data analysis cumbersome.

Another modification to the program would be the ability to add loading characteristics in order to achieve more accurate fatigue calculations. This would allow for other uses of the spring besides simple compression, such as use in vibrations mounts, or springs designed to absorb shocks, since these do not have a load that is evenly applied to the spring surface, which increases fatigue. This addition would also aid in the process of spring optimization since the designer would have a better idea of the true values of the stresses.

The current program will display only one result from all the possible results. In addition this result is the first result that the program finds. The program could be modified to provide several alternative possible designs, and select a representative sample of the available designs. This would provide even greater flexibility and save even more time and effort for the spring designer.

Finally, it should be achievable to solve the problem of obtaining a solution with a number of coils which is practical for the spring designer to produce. To accomplish this the number of coils suggested should have only whole, half, or quarter coils. This is a tolerance that is acceptable for nearly all the spring manufactures and the automatic coiling machines in use today.

Though the study of springs and their properties is a well established area, there is no real criterion for the evaluation of a design. In general, optimization has not beet used by the spring makers to choose among possible designs, as little information on what constitutes an optimal design exists and no formal way of deciding which of two designs is a better use of resources. The design and fabrication of springs is often driven by constraints, but this should not mean that there is no formal way to establish a more efficient design. With the wide use of springs and mass production techniques this should be a valuable procedure to have. It would be worth the effort to study the relationship between the volume of a material and spring performance. If a correlation is found it could allow the designer to know the minimum material required to achieve the design requirements.

REFERENCES

Adams, W. A., 1850, “Railway Carriage and Wagon Springs”, Instn. Mech. Engrs. Proc., Jan., p 19-31

Anonymous, 1858, “New Construction of Railway Springs”, Instn. Mech. 8 –Proc., p 160-165

Bannantine, Julie A, 1990, Fundamentals of Metal Fatigue Analysis,. p.2 and 5, Prentice Hall, Englewood Cliffs, N. J.

Begtrup, J., 1892, “Spiral Springs”, Am. Mach., v 15, n 33, p 2-3

Bergstrasser, M, 1933, “Die Berechnung Zylindrischer Schraubenfedern”, Z. VDI, Bd 77, 198.

Carlson, H., 1969, “Fatigue Testing of Springs”, Springs, May, p. 18-19

Carlson, H., 1985, “The Stress Correction Factor-When and When not to Use it”, Springs, October, p. 47-53

Caspari, E., 1875, “Isochronism of Spiral Watch Springs”, Academie des Sciences – Comptes Rendus. V 81, p 1122- 1123

Doubler, J. W. H., 1876, “Improved Spring Motor”, Sci. Am. Supp., v 2, p 791

Finniecome, J. R., 1947, “Critical Analysis of Stresses in Helical Springs of Circular Cross Section”,. Mech. World, v. 121, n. 3149, 3150, 3151, 3152, 3153, May 23, 1947, p. 4937, May 30, p. 523-7, June 6, p. 558-63, June 13, p. 585-7, June 20, p. 610, 612

Fordsham, C., 1847, “On the Laws of Isochronism of the Balance Spring as Connected with the Higher Order Adjustments of Watches and Chronometers”, Instn. Civil Engrs. Proc., p. 224-254

French, R. A., 1902, “Experiments on Spiral Springs”, Am. Soc. Mech. Engrs.–Trans., v 23, p 298-312

Gilbert, D., 1825, “General Nature and Advantages of Wheels and Springs for Carrying the Draft of Cattle and the Form of Roads”, Journal of Science, v. 18, p. 95-98

Goener, O., 1932, Die Berechnung zylindrischer Schraubenfedern, Z. VDI, Bd76, 269

Honegger, E., 1930, “Zur Berechnung von Schraubenfedern mit Kreisquerschnittt”, P. Appl. Mech., Vol. 12, 99. also “Calculation of Helical Springs of Round Wire”, Brown Boveri Rev., vol. 18, no. 35 Mar. 1931, p 120-125

Hooke, R., 1678, De Potentia Restitutiva, London

Lagrange, J. L., 1770, “Sur la Force des resorts plies”, Memoires de l’Academie de Berlin, XXV, Berlin.

Leveaux, E. H., 1876, “Spring Motors”, Sci. Am. Supp., v. 2, p 741-743

McCarty, 1898, “Coil Springs for Freight Cars”, Railroad Gaz., v 30, p119-120

Morandiere, J., 1866, “Belleville Steel Disk Springing System”, Societe des Ingenieurs Civils de France, p 629-642

Nadal, M. J., 1896, “Theory of the Stability of Locomotives”, Annales des Mines, ser 9, v 9, p 415-467

Philips, M., “Adjustable Spiral for Chronometers and Watches”, Annales des Mines: Memoires, v 20, ser 5, p 1-107

Rankine, W. J. M., 1866, “Elasticity of Spiral Springs”, Engineer, v 222, p 374

Resal, H, 1888, “The theory of the Belleville Spring”, Academie des Sciences-Comptes Rendus, v 107, p 713-718

Resal, H, 1892, “Properties of the Loxodromics of a Revolving Cone and it’s a Application to conical Springs”, Academie des Sciences – Comptes Rendus, v. 114, p 147-152

Rey, L., 1876, “Spring Suspensions of Railroad Equipment”, Societe des Ingenieurs , Civils de France, p 845-855

Roever, 1913, A., “Beanspruchung zylindrischer Schraubenfedern mit Kreisquerschnitt”, Z. VDI, Bd57, 1906.

Sweiskowski, H. B., 1995, “Max Out on Minimum Values: Designing Helical Compression Springs for Minimum Volume and Weight”, Springs, September 1995, p 39-59

Wahl, A. M., 1929, “Stresses in Heavy Close Coiled Helical Springs”, Transactions ASME v 51, also Am. Soc. Mech. Engrs., Advance Paper no. 39 for mtg. Dec. 3-7, 1928

Wahl, A. M., 1939, “Analysis of Effect of Wire Curvature on Allowable Stresses in Helical Springs”. Am. Soc. Mech. Engrs.-Trans. (J. Applied Mechanics), v. 6, n. 1 and 4, p. A-25-30

Wahl, A. M., 1944, Mechanical Springs 1st edition, McGraw-Hill, New York.

Wahl, A. M., 1963, Mechanical Springs 2nd edition, McGraw-Hill, New York.

Wolansky, E. B., 2005, “Lateral Deflection of Free-End Springs”, Springs, January, p. 33-34.

Young, A., 1852, “Theory of the Main Springs of a Watch Showing how to Select and Adjust One without Having to Try it in the Barrel”, Jl. Franklin Inst., v 54, p 344-347

Zubek, L., 2010, “Showcasing ASD 7.0: Spring in Series”, Springs, Winter, p. 55-56

BIBLIOGRAPHY

Cain, Tubal, 1988, Spring Design and Manufacture, Argus Books, Swanley, Kent, England.

Spring Manufacturers Institute, 2002, Handbook of Spring Design, Oak Brook, Il.

Society of Automotive Engineers, 1996, Spring Design Manual, Second Edition, Warrendale, Pa.

Wahl, A. M., 1991, Mechanical Springs, Spring Manufacturers Institute, Oak Brook, Il.

APPENDICES

Appendix A Program Code

A.1. Main Program

Rule
DATE=DATE()
TIME=TIME()
;Setting section
set_check = (Ss*Kw - Sys('unset)*Sut)/(Sys('unset)*Sut)*100
If and (set_check >= 10, set_check<= 30) then OK = 'yes else OK = 'no
If OK = 'yes then f = 'try_setting
FinalLoadChk=(KwSf-Sys)/Sys*100
;Error section
if and(solved(),FinalLoadChk > -10) then call errmsg("Final working Load is close to causing the spring to yeild, consider a new design.")
IF AND(solved(),solid/fwh=>1,SOLVED()) THEN CALL ERRMSG("SPRING WILL GO SOLID")
IF d=>od/2 THEN CALL ERRMSG("WIRE DIA TOO LARGE, OR SPRING DIA TOO SMALL")
if and(BuckleFlag=1,B=1) then call ErrMsg("Spring may not be stable with given end condition . Try to use a 1/4 or 3/4 turn for the ends.")
IF and (solved(),set_check > 35,setting ='set,FinalLoadFlag=1) THEN CALL ERRMSG("Setting spring causes too much stress. Change setting to 'unset'.")
;Peening section
Sew = TEL(peen)
If (peen = 'unpeened) then Sfw = Sfw_unpeen(cycles)*Sut
If (peen = 'peened) then Sfw = Sfw_peen(cycles)*Sut
;Spring design equantions

Appendix A.1. (Continued)

od=id+2*d
D=od-d ; MEAN DIA
c=od/d-1 ; SPRING INDEX
delta_h=iwh-fwh ; WORKING STROKE
rate=(Lf-Li)/(delta_h) ; RATE BASED ON LOAD AND DEFLECTION
free=iwh+Li/rate ; FREE LENGTH
if EndCon =1 then solid=Nt*d else solid = (Nt+1)*d ; modified to include end conditions
Ls=rate*(free-solid) ; SOLID LOAD ; note this equation uses a DEFELCTION so end condidtions are irrelevant
L3 = rate*(free-h3) ;Load at specified height
rate=G*d^4/(8*D^3*Na)
Nt=Na+Ni
L100 = (free - h100) *rate
Fn=14000*d/(Na*D^2) ; Approx natural frequency (Lee spring catalog)
p=(free-2*d)/Na ;calculates pitch Note: Makes no alowece for end conditions
ODF=(D^2+(p^2-d^2)/(pi())^2)^0.5+d ; Solid hight outer diameter
;Stress equations
Kw=(4*c-1)/(4*c-4)+.615/c ; WAHL STRESS CONCENTRATION FACTOR
Ks=1+.5/c
Si=8*Li*D/(PI()*d^3) ; STRESS AT INITIAL ;this equation was using a stress concentration factor for set springs. Why
Sf=8*Lf*D/(PI()*d^3) ; STRESS AT FINAL ;this equation was using a stress concentration factor for set springs. Why?
Ss=8*Ls*D/(PI()*d^3) ; STRESS AT SOLID ;this equation was using a stress concentration factor for set springs. Why
KwSi = CorrectedStress(Kw,Ks,Si,setting)
KwSf= CorrectedStress(Kw,Ks,Sf,setting)
KwSolid=CorrectedStress(Kw,Ks,Ss,setting)
S100=8*L100*D*Kf/(PI()*d^3) ; 100,000 psi

Appendix A.1. (Continued)

;Buckling equations
Buck_x = free/D
Buck_y = (free - fwh)/free
call Buckle(;buckdef,LB) ; buckling
BK1 = 1/(2*(1-G/BMod))
BK2 = 2*pi()^2*(1-G/BMod)/(1+2*G/BMod)
;Fatigue calculations
if Material =1 then Sut = 184649*d^(-.1625) else Sut=239000
Kf=Kw
Sus = 0.67 * Sut
Sa=8*Fa*D*Kw/(PI()*d^3) ; alternating stress
Fa = (Lf - Li)/2
Fm = (Li + Lf)/2
Sm=8*Fm*D*Ks/(PI()*d^3) ; mean stress
Ses = (0.707*Sew*Sus)/(Sus - 0.707*Sew)
Sfs = (0.707*Sfw*Sus)/(Sus - 0.707*Sfw)
Nfs_finite_check = Sfs*(Sus -Kw* Si)/(Sfs*(Sm - Kw*Si) + Sus*Sa)
Nfs_infinite_check = Ses*(Sus - Kw*Si)/(Ses*(Sm – Kw*Si) + Sus*Sa)
Sys = Sys(setting)*Sut
Ns_shut = Sys/(Ks*Ss)
Ns = Sys/(Ks*Sf)
Sms = Sus*(Sfs^2 - Sfs*Sa + Sus*Sm)/(Sfs^2 + Sus^2)
Sas = -Sfs/Sus*Sms + Sfs
ZS = sqrt((Sm - Sms)^2 + (Sa - Sas)^2)
OZ = sqrt(Sa^2 + (Sm - Kw*Si)^2)
Nf2 = (OZ + ZS)/OZ
Sms_i = Sus*(Ses^2 - Ses*Sa + Sus*Sm)/(Ses^2 + Sus^2)
Sas_i = -Ses/Sus*Sms_i + Ses
ZS_i = sqrt((Sm - Sms_i)^2 + (Sa - Sas_i)^2)
OZ_i = sqrt(Sa^2 + (Sm - Kw*Si)^2)
Nf2_i = (OZ_i + ZS_i)/OZ_i
Tms = (Sys + Sm - Sa)/2
Tas = (Sys + Sa - Sm)/2

Appendix A.1. (Continued)

ZSy = sqrt((Sm - Tms)^2 + (Sa - Tas)^2)
Nfy = (OZ + ZSy)/OZ
If Ns < 1 Then Nfs_finite = f Else Nfs_finite = MIN(Nf2,Nfy,Nfs_finite_check)
If Ns < 1 Then Nfs_infinite = f Else Nfs_infinite = MIN(Nf2_i,Nfy,Nfs_infinite_check)
;These equations were added into the program form other sources
if solved() then call GetStandard(d,Material;Dws,Dwl,WireAvailability) ;ASD
Lw = Lw() ; Length of Wire required to form the Spring ;ASD
Ap = ATAN(0.5 * p / (D * PI() / 2)) * 180 / PI() ;Pitch Angle ASD
Def=free-solid ;Available Deflection
Na = G*d^4/(8*rate*D^3) ;Number of Active coils ASD
FWP=Lf/((Pi()*DIA1^2*.25)-(Pi()*DIA2^2*.25)) ;imported from spring2 calculates pressure
IWP=Li/((Pi()*DIA1^2*.25)-(Pi()*DIA2^2*.25)) ; imported from spring2
;JP's equns
LinearRatio = (free-fwh)/(free-solid) ; raito for linearity
if Material = 1 then G = given('G,G,11500000) else G = given('G,G,11000000) ;imported from spring2 gets a modulus based on wire dia, or the modulus of 17-7 stainless steel
if Material = 1 then BMod=given('BMod,BMod, 3E7) else BMod=given('BMod,BMod,2950000) ; gives default values for music wire or stainless steel
cycles = given('cycles, cycles, 10E6) ; gives default values for infinite life test.
Call ChrisGoodman(Ses,Sew,Sus,Sfs,Sfw,Si,Sm,Sa,Sys)
Call BlankASDPlot() ; essentially from ASD rule sheet Just in a function now
DIA2=given('DIA2,DIA2,0)
if EndCon =1 then Ni=given('Ni,Ni,2.5) else Ni=given('Ni,Ni,3) ;default number of inactive coils is 2.5 CG and 3C
Material=given('Material,Material,1)
S100=given('S100,S100,100000)
EndCon=given('EndCon,EndCon,1)
if solved() then Li=given('Li,Li,0.00000000000001)

Appendix A.1. (Continued)

YeildHT=-(Sys('unset)*Sut*PI()*d^3)/(Kw*8*D*rate)+free
SetHT=-(Sys('unset)*Sut*PI()*d^3)/(Kw*8*D*rate)*1.3+free
BC=given('BC,BC,4)
if given('d) then d=d else call Guess_Wire_dia(Solver;DiaWireTest,d,ODTest) ;guesses the wire dia
if buckdef=("No Buckle") then Buck=Def else Buck =buckdef ; this set of equations are ment to reduce chance
if and((Buck)/(Def)
if Mod(Nt*10,5)=0 then B=1 ; effective this is so it is canceled by me but can be activeated if it is found to be accurate.
Solver=given('Solver,Solver,2) ; . Default for the solver.
Special_Set=given('Special_Set,Special_Set,"no")
if Special_Set="yes" then FinalLoadFlag=0 else FinalLoadFlag=1
;Spring2 Plots
if or(Li=0,Material=2) then call blank('life_inf)
if or(Li=0,Material=2) then call blank('life_100k)
if or(Li=0,Material=2) then call blank('Ap_factor)
if and (Lf > Li,Material=1) then call InfinateLifeII(Lf,Li,KwSf,d;life)
UDP=L3/((Pi()*DIA1^2*.25)-(Pi()*DIA2^2*.25)) ;imported from spring2 calculates pressure
EndCond=given('EndCond,EndCond,1) ;Default is Ground ends
;ASD plots
if solved() then call FillLists(;ZZZ)
if solved() then call Fill1(Li,iwh,P1t)
if solved() then call Fill2(Lf,fwh,P2t)
P1t = Toload(Li) ; load tolerances
P2t = Toload(Lf) ; load tolerances

Appendix A.1. (Continued)

Lft = Tolfree(rate)
KE = CE*N^Y
GE = 386
if (setting='set) then call CYCLES(;KE,CE,Y) else N = "N/A"
if solved() then call Goodman()
if solved() then call holeshaft(ShaftDia,id,od,HoleDia,ODF)
;Spring2 plots

A.2. Solver Subroutine

Statement
ExitVar=0
LoopFlag=0
if Material=1 then Wires=145 else Wires=114
for x=1 to Wires
if Pre=0 then ExitVar = 1
If Pre=0 then exit
if given('d) then exit
if x=Wires then LoopFlag=LoopFlag+1
If and(x=Wires,LoopFlag=1) then x=1
if x=Wires then d=0
if x=Wires then call errmsg("The guess function can not find a suitable wire diameter to make a spring with the parameters given. You may want to change the parameters or try a larger wire diamter then .283 in.")
If Material=1 then d='New_Spring_Guess_list[x]
if Material=2 then d= 'WDS2[x]
if Pre= 1 then VarDia=.00 else VarDia=.0
if Pre=1 then DiaCount=1 else DiaCount=1
ODI=.001
ODTest=od-VarDia
D=ODTest-d
for y=1 to DiaCount
Del_h=iwh-fwh
rate=(Lf-Li)/Del_h

Appendix A.2. (Continued)

Na=(G*d^4)/(rate*8*D^3)
Nt=Na+Ni
solid=Nt*d
free=iwh+Li/rate
LinRatio=(free-fwh)/(free-solid)
Ls=rate*(free-solid)
Ss=Ls*8*D/(Pi()*d^3)
Sf=Lf*8*D/(Pi()*d^3)
C=ODTest/d-1
if and (C<=1,LoopFlag=0) then C=1.01
if C=1 then C=1.01
Ks=1+.5/C
Kw=(4*C-1)/(4*C-4)+.615/C;
Sut=Ultimate_Stress(d,Material)
if LoopFlag=0 then Sys=Sys('unset)*Sut else Sys=Sut*Sys('set)
if LoopFlag=0 then Stress=(Kw*Sf-Sys)/Sys else Stress=(Ks*Sf-Sys)/Sys
Set_check = (Ss*Kw - Sys('unset)*Sut)/(Sys('unset)*Sut)*100
IF and(set_check > 35,set_check
if Pre=2 then R=.9 else R=.85
if Pre=2 then W=1.1 else W=1.15
if Pre=2 then S=-.1 else S=-.3
if and(LinRatioW,Stress
If Pre =1 then goto Skip
;this block will look at fatigue charicteristics
Si=8*Li*D/(PI()*d^3) ; STRESS AT INITIAL ;now uses the Whal concentration factor
Fa = (Lf - Li)/2
Fm = (Li + Lf)/2
Sus = 0.67 * Sut
if peen='unpeened then Sfw=Sfw_unpeen(cycles)*Sut
if peen= 'peened then Sfw =Sfw_peened(cycles)*Sut
Ns = Sys/(Ks*Sf) ; static safety
Sa=8*Fa*D*Kw/(PI()*d^3) ; alternating stress
Sm=8*Fm*D*Ks/(PI()*d^3) ; mean stress
Sfs = (0.707*Sfw*Sus)/(Sus - 0.707*Sfw)

Appendix A.2. (Continued)

Nfs_finite_check = Sfs*(Sus - Kw*Si)/(Sfs*(Sm - Kw*Si) + Sus*Sa)
Sms = Sus*(Sfs^2 - Sfs*Sa + Sus*Sm)/(Sfs^2 + Sus^2)
Sas = -Sfs/Sus*Sms + Sfs
OZ = sqrt(Sa^2 + (Sm - Kw*Si)^2)
ZS = sqrt((Sm - Sms)^2 + (Sa - Sas)^2)
Nf2 = (OZ + ZS)/OZ
Tms = (Sys + Sm - Sa)/2
Tas = (Sys + Sa - Sm)/2
ZSy = sqrt((Sm - Tms)^2 + (Sa - Tas)^2)
Nfy = (OZ + ZSy)/OZ
If Ns < 1 Then Nfs_finite = f Else Nfs_finite = MIN(Nf2,Nfy,Nfs_finite_check)
If Pre=1 then Finite_life= 1.2 else Finite_life=1.1
if Nfs_finite>Finite_life then ExitVar = 1 else ExitVar = 0
DiaWireTest:= d
if ExitVar=1 then exit
ODTest=ODTest+ODI
Skip:
Next y
if ExitVar=1 then exit
Next x

Appendix B Governing Equations

Load

$$P = \frac{\pi d^3 \tau}{8D}$$

Stress

$$\tau = \frac{\delta G d}{\pi D^2 n}$$

$$\tau' = K\tau = \frac{8PDK}{\pi d^3}$$

Wahl correction factor

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

Deflection

$$\delta = \frac{\pi D^2 n \tau}{Gd}$$

$$\delta = \frac{8PD^3 n}{Gd^4}$$

Spring Rate

$$k = \frac{P}{\delta} = \frac{Gd^4}{8D^3 n}$$

Spring Index

$$c = \frac{D}{d}$$

Appendix C Test Data

C.1. Calibration Data

C.1.1. 100 lb Sensor

****

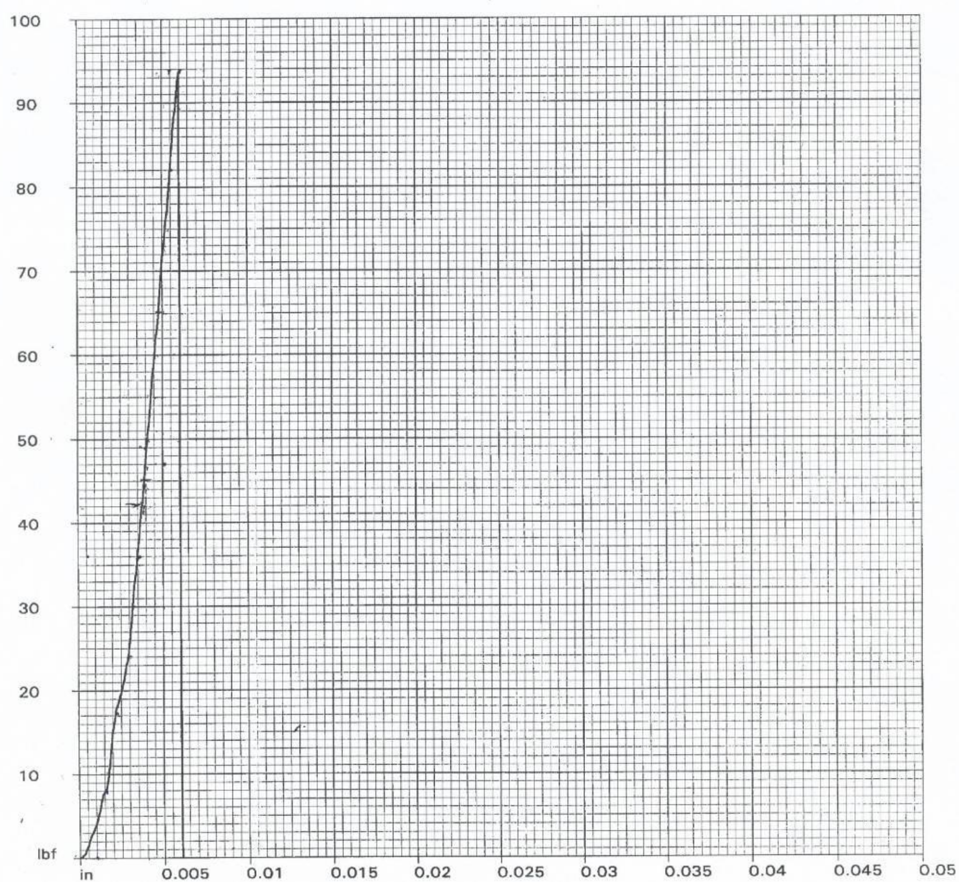
SUN HYDRAULICS

Test Report

****

Product Code ..:DEFLECTION
Date ....:1/28/03
Batch Number ..:
Operator ......:DWS
Test Speed.....: 0.100{in/min}

	Max lbf	Break lbf	Ext @ Brk in
1	-94.232	-93.445	-0.005

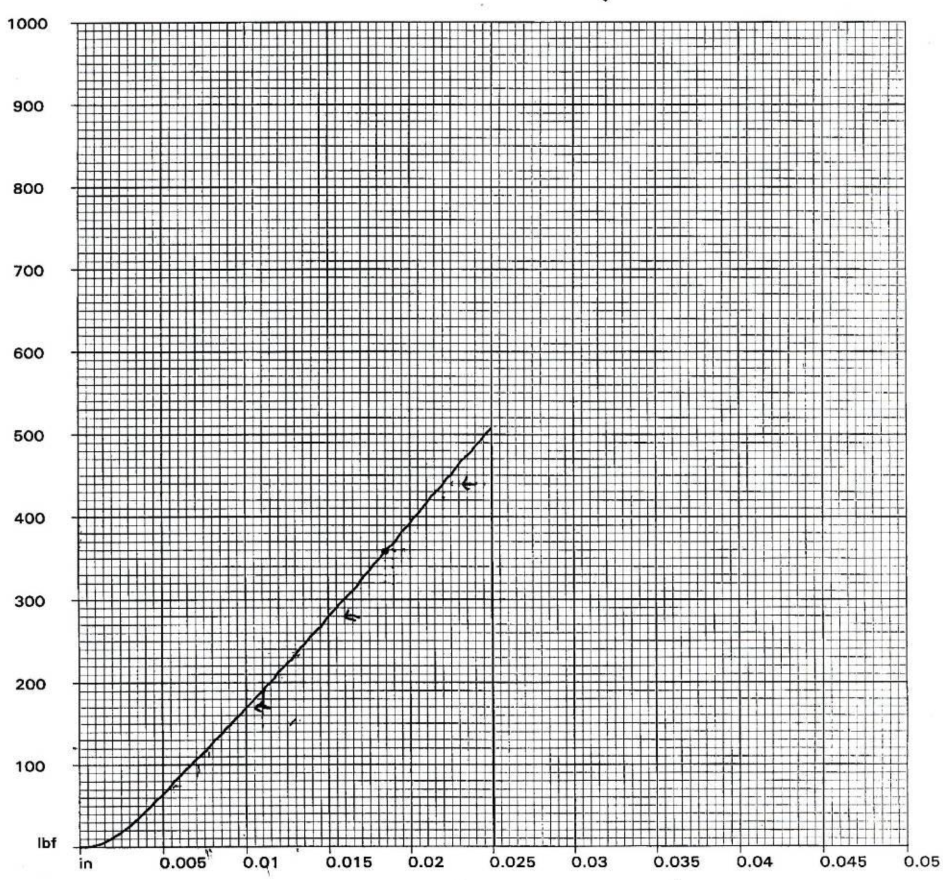


Appendix C.1.2. 1000 lb Sensor

```
****************
SUN HYDRAULICS
FORCE/DISTANCE
****************

Product Code ..:DEFLECTION
Date ..........:03/01/06
Batch Number ..: 1000 POUND
Operator ......:DWS
Test Speed.....:     0.500{in/min}
```

	Max lbf	Break lbf	Ext @ Brk in
1	-512.376	-510.390	-0.025



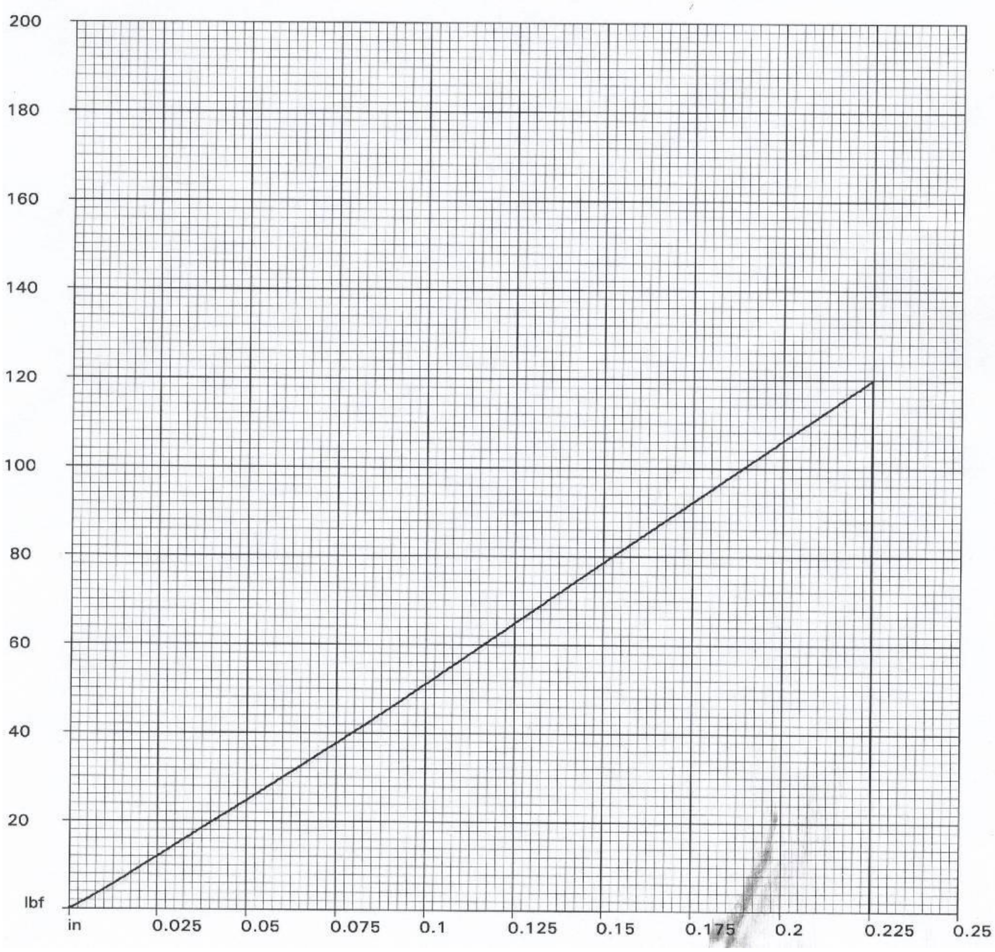
Appendix C.2. Sample Spring Plots

C.2.1. Spring 225

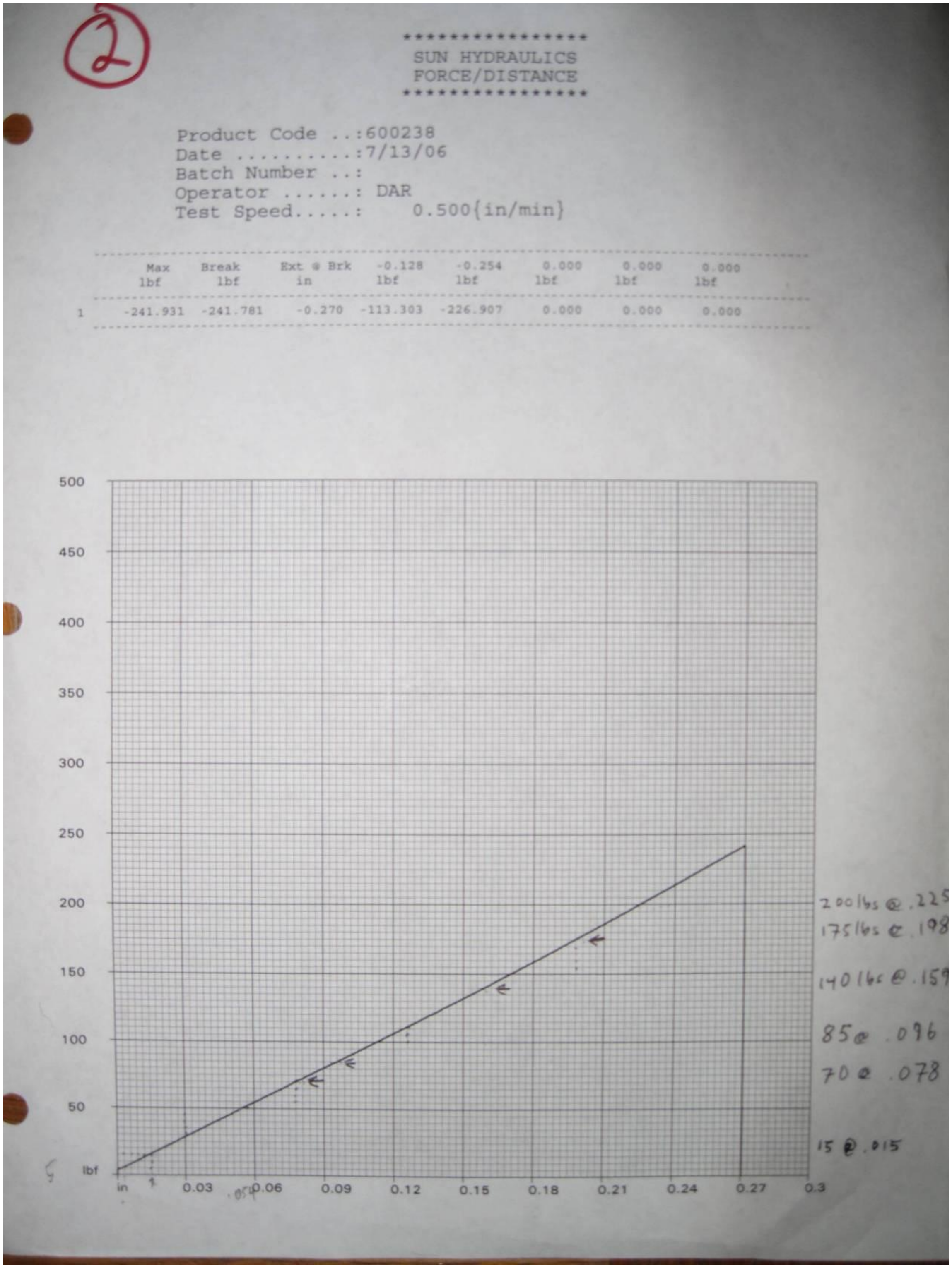
****
SUN HYDRAULICS
FORCE/DISTANCE
****

Product Code ..:600225
Date ....:7/17/06
Batch Number ..:
Operator ......: DWS
Test Speed.....:   0.500{in/min}

	Max lbf	Break lbf	Ext @ Brk in	-0.133 lbf	-0.211 lbf	0.000 lbf	0.000 lbf	0.000 lbf
1	-120.085	-120.047	-0.225	-69.390	-112.404	0.000	0.000	0.000



Appendix C.2.2. Spring 238


SUN HYDRAULICS
FORCE/DISTANCE

Product Code ..:600238
Date ....:7/13/06
Batch Number ..:
Operator ......: DAR
Test Speed.....: 0.500{in/min}

	Max lbf	Break lbf	Ext @ Brk in	-0.128 lbf	-0.254 lbf	0.000 lbf	0.000 lbf	0.000 lbf
1	-241.931	-241.781	-0.270	-113.303	-226.907	0.000	0.000	0.000

Appendix C.3. Sample Excel Data Sheet

600-238

Initial force offset
4

Force	defection	Spring rate	Correction	Correced fo	Corrected r	Linear Correction eqn.
20	0.021	761.9048	0.001007	20.76746	798.4505	0.001007
95	0.108	862.069	0.004785	99.12468	900.6577	0.004785
112.254	0.128	862.7	0.005654	117.1314	900.3349	0.005654
155	0.177	872.3673	0.007807	161.8101	908.4849	0.007807
185	0.21	909.0909	0.009317	193.4704	959.4019	0.009317
215	0.243	909.0909	0.010828	224.844	950.7145	0.010828
225.408	0.254	946.1818	0.011353	236.1496	1027.783	0.011353
240.358	0.27	934.375	0.012106	251.6691	969.9688	0.012106
		882.2225			926.9746	

A	B	W1	W2	CR1	CR2
-9.05E-08	7.47E-05	112.254	225.408	0.00725	0.01225

Corrected f	Corrected r	Quandratic corr
21.11138	814.8275	0.001459
100.4172	911.5614	0.006284
118.5086	904.568	0.00725
163.21	910.0407	0.009411
194.755	955.9078	0.01073
225.8062	940.9474	0.011887
236.9987	1017.498	0.01225
252.2594	953.7926	0.012737
0		0
	926.143	